Fast Accurate Analytical Shear Estimation

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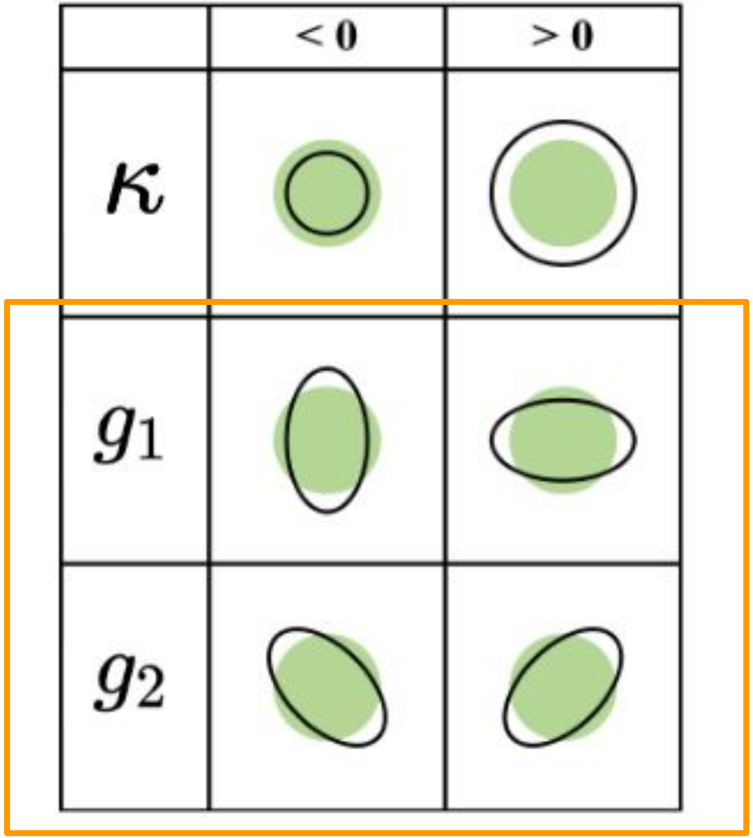
Shear Distortion

$$f_I(\vec{x}_I) = \boxed{f(\vec{x})}$$ galaxy

$$\vec{x}_I = \mathbf{A}\vec{x}$$

$$\mathbf{A} = \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}$$

where $g_{1,2} \ll 1$



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How to measure the shear distortion from galaxy images?

(assuming intrinsic galaxies are randomly oriented)

Shear response and Metacalibration

Observable **O**: projection of galaxy onto a basis —

$$O = \int f(\vec{x})\chi(\vec{x})d^2x$$

$$= \int f_I(\mathbf{A}\vec{x})\chi(\vec{x})d^2x$$

How does **O** transform under shear distortion?

Huff & Mandelbaum (2017), Sheldon & Huff (2017):

$$O = O_I + g_1 \boxed{\frac{\partial O}{\partial g_1}} + g_2 \boxed{\frac{\partial O}{\partial g_2}}$$

$$\boxed{\frac{\partial O}{\partial g_i}} = \frac{\partial\left(\int f_I(\mathbf{A}\vec{x})\chi(\vec{x})d^2x\right)}{\partial g_i}$$

shear response can be estimated by adding small shear distortions to galaxy image.

Another line of thought

$\vec{x} \to \mathbf{J}\vec{x}$

$\mathbf{J} = \mathbf{A}^{-1}$

$$O = \int \boxed{f_I(\mathbf{A}\vec{x})}\chi(\vec{x})d^2x$$

$$O \propto \int f_I(\vec{x})\boxed{\chi(\mathbf{J}\vec{x})}d^2x$$

If we ***fix the galaxy images and distort the basis***, the measurement shall be **faster**, since we can precompute the **shear response of the basis**.

$$\frac{\partial O}{\partial g_i} \propto \int f_I(\vec{x})\boxed{\frac{\partial \chi(\mathbf{J}\vec{x})}{\partial g_i}}d^2x$$

Chain rule for derivatives

$$\frac{\partial \chi(\mathbf{J}\vec{x})}{\partial g_1} = \left(x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}\right)\chi(\vec{x})$$

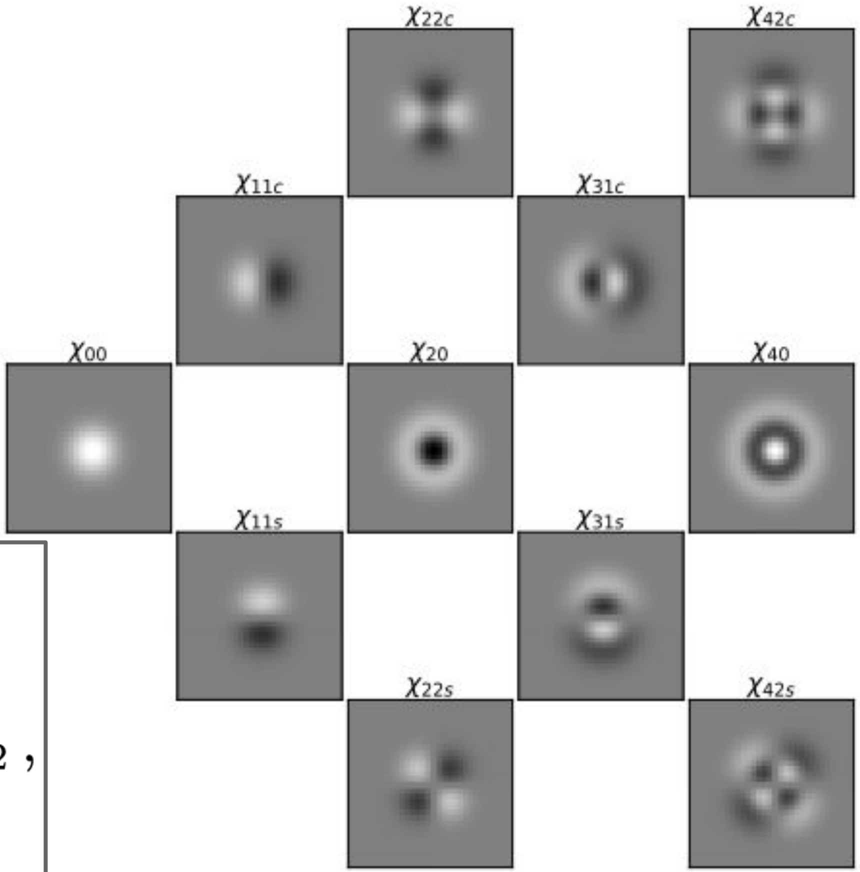
$$\frac{\partial \chi(\mathbf{J}\vec{x})}{\partial g_2} = \left(x\frac{\partial}{\partial y} + y\frac{\partial}{\partial x}\right)\chi(\vec{x})$$

Shaplets 1: shapelets – eigen functions of 2D QHO

Massey & Refregier (2005):

$$M_{nm} = \int f(\vec{x}) \chi_{nm} d^2 x$$

- ‘n' determines **radial profile**;
- ‘m' determines **spin**;
- ‘c' refers to **real part** (cosine);
- ‘s’ refers to **imaginary part** (sine).



$$\frac{\partial \chi_{n,m}(\mathbf{J}\vec{x})}{\partial g_1} = a_1 \chi_{n-2,m-2} + a_2 \chi_{n+2,m-2} + a_3 \chi_{n-2,m+2} + a_4 \chi_{n+2,m+2} \, ,$$

where a_i is function of (n, m)

Shaplets 2: shear estimation

Massey & Refregier (2005):

$$M_{22c} = M^I_{22c} + \frac{\sqrt{2}}{2}\gamma_1 (M^I_{00} - M^I_{40}) - \sqrt{3}\gamma_1 M^I_{44c} - \sqrt{3}\gamma_2 M^I_{44s},$$
$$M_{22s} = M^I_{22s} + \frac{\sqrt{2}}{2}\gamma_2 (M^I_{00} - M^I_{40}) + \sqrt{3}\gamma_2 M^I_{44c} - \sqrt{3}\gamma_1 M^I_{44s}$$
$$\langle M_{00} - M_{40}\rangle = \langle M^I_{00} - M^I_{40}\rangle$$

Two good things ☺️:

- Shear only couples a **finite number** of shaplet basis (thus modes).
- Shapelet bases are **orthogonal** so that we do not need to know other modes when estimating one specific modes.

Shaplets 2: shear estimation

Massey & Refregier (2005):

If there is no systematics.

$$\hat{g}_1 = \frac{\langle M_{22c} \rangle}{\langle M_{00} - M_{40} \rangle}$$

$$\hat{g}_2 = \frac{\langle M_{22s} \rangle}{\langle M_{00} - M_{40} \rangle}$$

Two good things ☺️:

- Shear only entangles a **finite number** of shaplet basis (thus modes).
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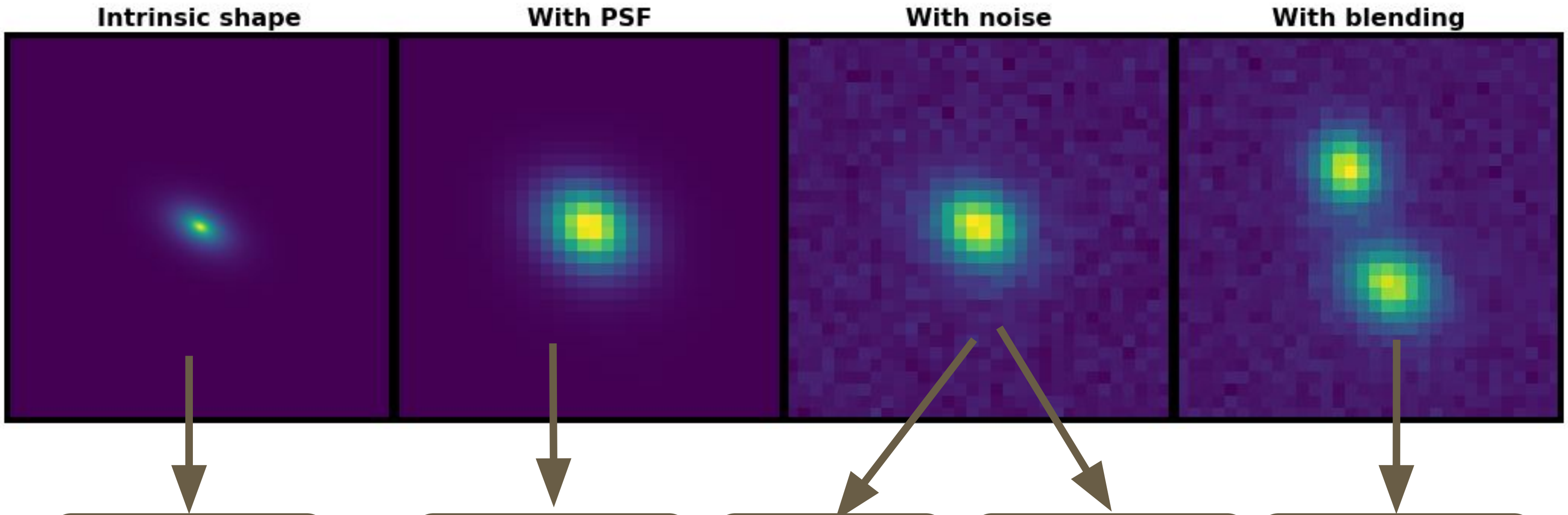
Systematics in weak lensing shear measurement



Shape noise: O_I

PSF correction

noise bias

measurement error: δO

Future works

$$\hat{g}_{1,2} = (1 + \boxed{m})g_{1,2} + \boxed{c_{1,2}}$$

multiplicative bias

additive bias

Power of Fourier Transform

Zhang (2008)

$$\tilde{f}_o(\vec{k}) = \int f_o(\vec{x}_o) e^{-i\vec{k}\vec{x}_o} d^2 x_o,$$

$$\tilde{F}_o(\vec{k}) = |\tilde{f}_o(\vec{k})|^2 .$$

Li et. al (2018)

$$M_{nm} = \int \chi^*_{nm} \frac{\tilde{F}_o - \tilde{N}}{\tilde{G}} d^2 k.$$

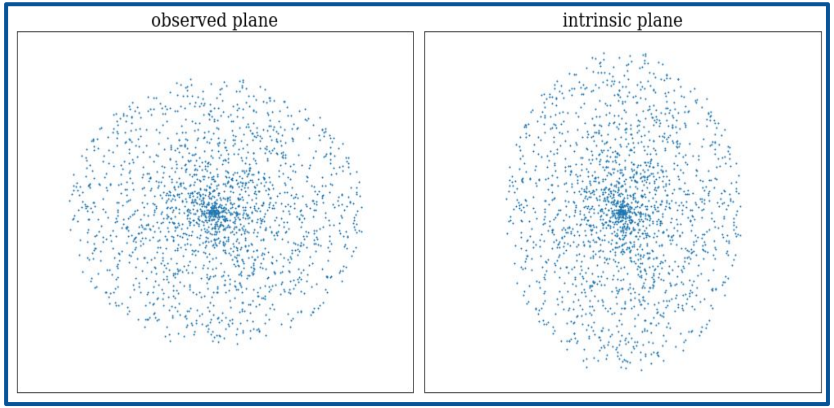
PSF deconvolution *Why Fourier space?*

$$f_o(\vec{x}_o) = \int f(\vec{x}) g(\vec{x}_o - \vec{x}) d^2 x$$

$$\tilde{F}(\vec{k}) = \frac{\tilde{F}_o(\vec{k})}{\boxed{\tilde{G}(\vec{k})}}$$

Fourier power of PSF

Offcentering effect *Why use power?*



Normalization

Li et. al (2018)

$$e_1 = \frac{M_{22c}}{M_{00}+\boxed{C}}, \qquad e_2 = \frac{M_{22s}}{M_{00}+C}.$$

$$R_i = \frac{\partial e_i}{\partial g_i} = \frac{\sqrt{2}}{2}\frac{M_{40}-M_{00}}{M_{00}+\boxed{C}} - \sqrt{2}e_i^2$$

$$\boxed{\hat{g}_i = \frac{\langle e_i \rangle}{\langle R_i \rangle}}$$

C changes the relative **weight** between galaxies and reduce nonlinear **noise bias**.

Noise bias correction

Li et. al (2022)

$$\boxed{\tilde{e}_1 = \frac{M_{22c}+\mathcal{E}_{22c}}{M_{00}+C+\mathcal{E}_{00}}}$$

$$\langle e^I \rangle = 0; \quad \langle \delta e \rangle = 0$$

$$\hat{e}_1 = \frac{1}{T}\left(\tilde{e}_1 + \frac{\boxed{\mathcal{V}_{0022c}}}{(\tilde{M}_{00}+C)^2}\right) + \mathcal{O}\left(\left(\frac{\mathcal{E}_{nm}}{M_{00}+C}\right)^4\right)$$

$$T = 1 + \frac{\boxed{\mathcal{V}_{0000}}}{(\tilde{M}_{00}+C)^2}$$

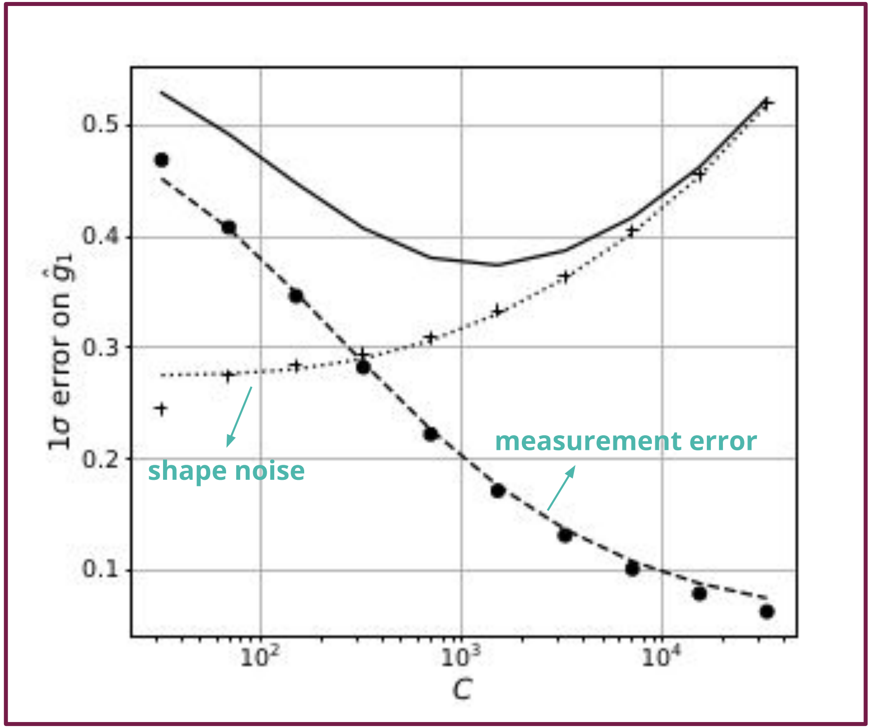
Covariance between measurement errors of shapelet modes, which can be derived if photon noise is homogeneous Gaussian field.

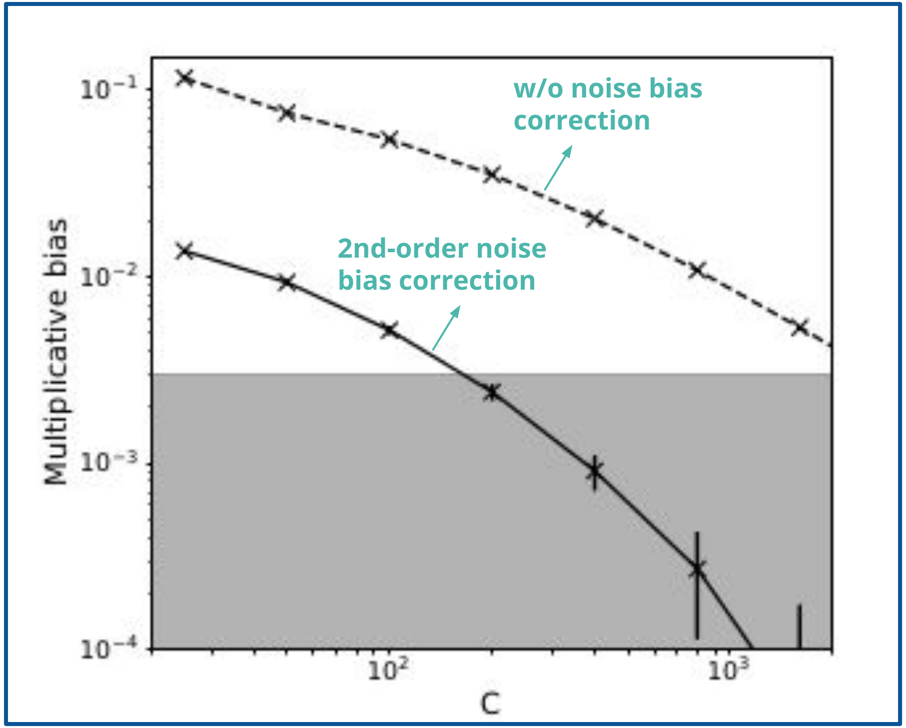
Noise-Bias tradeoff with C

Similar precision to reGauss (optimally weighted) for 24.5 magnitude limited galaxies with HSC-like observational conditions.

~0.001 cpu second to process one galaxy.
(metacal takes ~0.3 cpu second)
Multiplicative bias:

$$\hat{g} = (1+m)g + c$$



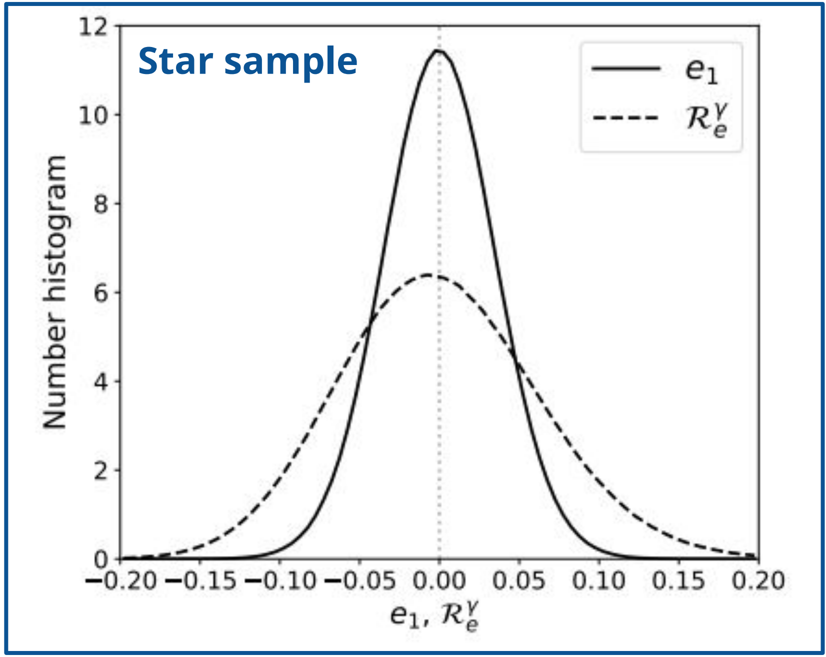


Small galaxies and Stellar contaminations

Small galaxies

Table 3. The average reGauss resolution (first row), shear multiplicative bias (second row), and shear additive bias (third row) for three samples of galaxies simulated as a collection of random point sources. All these galaxies are smaller than those in the HSC shape catalog, which analyses only galaxies with reGauss resolution greater than 0.3 (Li et al. 2021).

	smallest	smaller	small
reGauss resolution	0.12	0.19	0.27
$m_1(10^{-4})$	0.4 ± 0.9	-1.4 ± 0.7	-1.0 ± 0.5
$c_1(10^{-4})$	-2.8 ± 4.5	-0.8 ± 1.1	-0.5 ± 0.7



It reaches subpercent level accuracy even for extremely small galaxies and stellar contaminations will not bias the shear measurement.

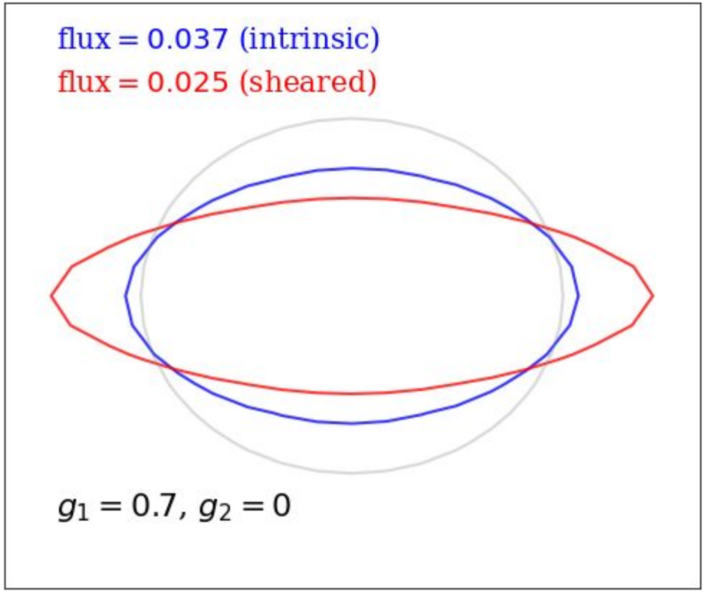
Selection bias 1 (Kaiser flow)

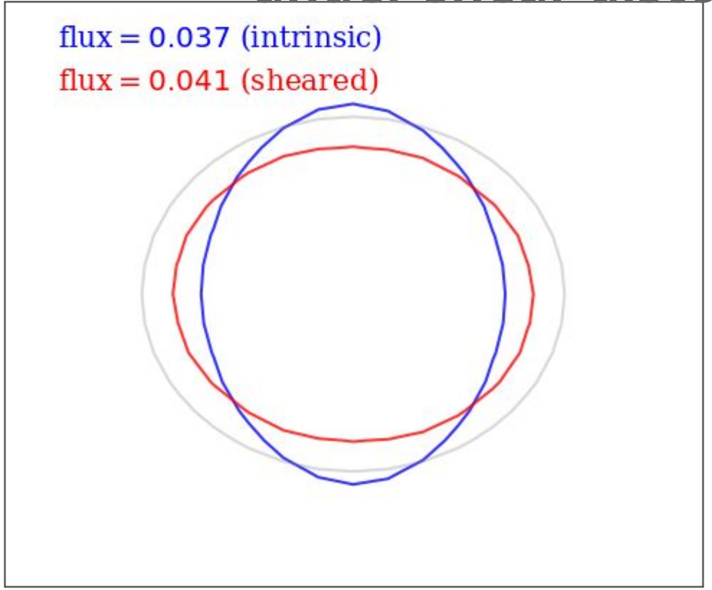
$$\langle e^I \rangle = 0;$$
$$\langle \delta e \rangle = 0$$

Kaiser (2000), Li et. al (2022)

$$\langle e^I_\alpha \rangle = -\gamma_\alpha \boxed{P(s)} \left\langle e_\alpha \frac{\partial s}{\partial \gamma_\alpha} \right\rangle \Big|_{s=s_{\rm low}}$$

Galaxies with exactly the same profile but different orientations have different measured flux under shear distortion.





Note, we assume:

$$P(s) \sim \tilde{P}(s)$$

Selection bias 2 (e-flux error correlation)

Li et. al (2022)

δe and δs denote measurement errors due to photon noise on ellipticity and flux. They average to zero if there is no selection.

The errors are correlated, since noise can be anisotropic in <u>the PSF deconvolved space.</u>

$$\langle \delta e_\alpha \delta s \rangle \neq 0$$

A selection on $\boxed{\mathrm{s} + \delta \mathrm{s}}$ results in $\langle \delta e_\alpha \rangle \neq 0\,.$

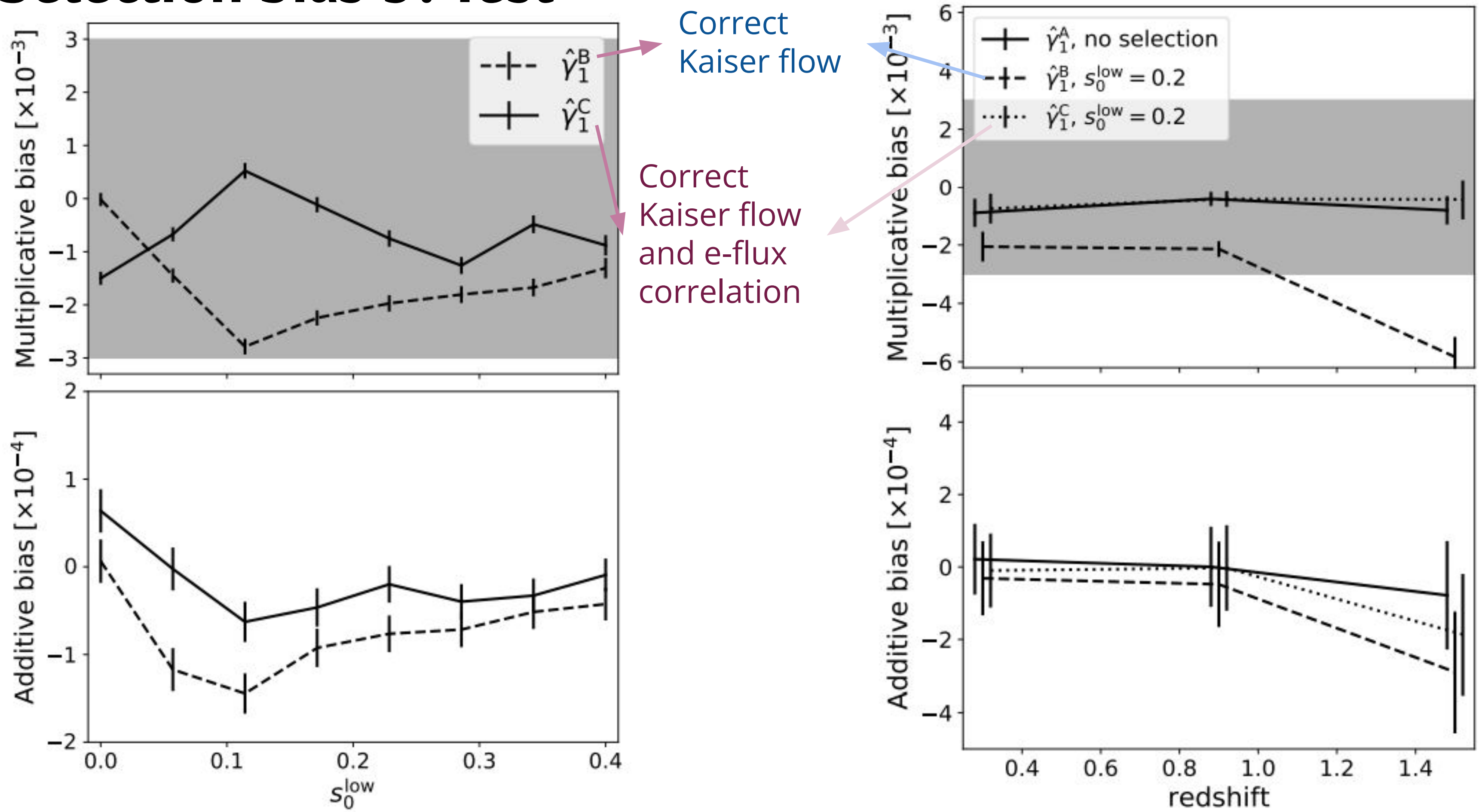
$$\hat{e}_1^{\Delta} \simeq \iint_{s_0^{\mathrm{low}}} e_1 \left(P \bigotimes \boxed{P^\delta} - P \right) \mathrm{d}e_1 \mathrm{d}s$$

PDF of measurement errors.

Note, we assume:

$$P(s) \sim \tilde{P}(s)$$

Selection bias 3: Test



Summary

1. Four shapelet modes to measure shear! 0.001 cpu seconds/ gals!!
2. Noise-bias tradeoff with parameter: C;
3. Covariance of measurement errors due to photon noise can be derived if noise is homogeneous Gaussian; and nonlinear noise bias can be revised;
4. Selection bias including Kaiser flow and e-flux errors correlation can be corrected taking an approximation that the difference between noisy PDF and noiseless PDF are small. Multiplicative bias can be controlled at subpercent level;
5. We are deriving the shear response of detection process.

Thanks!

Codes are available at https://github.com/mr-superonion/FPFS