Systematics in Weak Lensing

Shear Estimation

Xiangchong Li

The University of Tokyo

Weak Gravitational Lensing

Image Distortion

Denote the surface brightness distribution of a galaxy as $\mathrm{f}(\vec{\mathrm{x}})$

$$\begin{aligned}\vec{\mathrm{x}}_{\mathrm{l}} &= \mathbf{J}\vec{\mathrm{x}},\\ \mathrm{f}_{\mathrm{l}}(\vec{\mathrm{x}}_{\mathrm{l}}) &= \mathrm{f}(\vec{\mathrm{x}}),\end{aligned} \tag{1}$$

where the Jacobi matrix ($\mathbf{J}$) is

$$\mathbf{J} = \begin{pmatrix} 1+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\gamma_1 \end{pmatrix}. \tag{2}$$

$$|\gamma_{1,2}| \sim 0.02$$

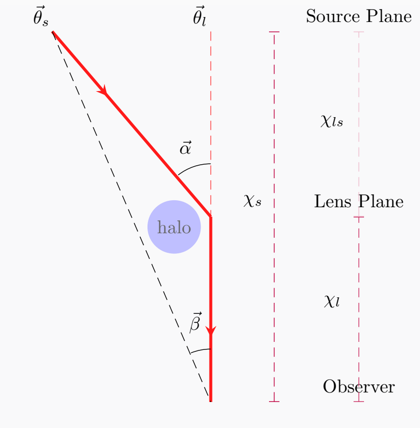


Image Distortion

Denote the surface brightness distribution of a galaxy as $f(\vec{x})$

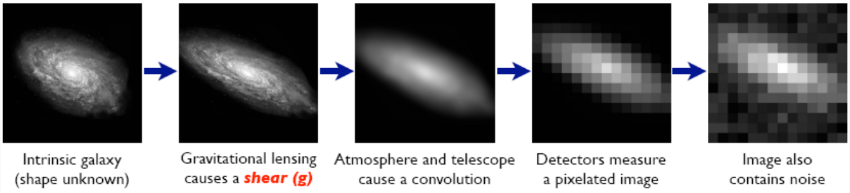
$$\begin{aligned} \vec{x}_l &= \mathbf{J}\vec{x}, \\ f_l(\vec{x}_l) &= f(\vec{x}), \end{aligned} \tag{3}$$

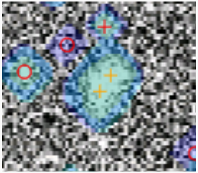
where the Jacobi matrix ($\mathbf{J}$) is

$$\mathbf{J} = \begin{pmatrix} 1+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\gamma_1 \end{pmatrix}. \tag{4}$$

$$|\gamma_{1,2}| \sim 0.02$$

Background source galaxies




Blending

Bias definition

$$\hat{\gamma}_{1,2} = (1 + m_{1,2})\gamma_{1,2} + c_{1,2}, \qquad (5)$$

where $m_{1,2}$ is multiplicative bias, $c_{1,2}$ is additive bias.

Requirement of future surveys

$$|m| < 0.5\% \qquad (6)$$

- Measure Shear distortion to sub-percent level accuracy

Image Distortion

Denote the surface brightness distribution of a galaxy as $\mathrm{f}(\vec{x})$

$$\begin{aligned} \vec{x}_l &= \mathbf{J}\vec{x}, \\ f_l(\vec{x}_l) &= f(\vec{x}), \end{aligned} \tag{7}$$

where the Jacobi matrix ($\mathbf{J}$) is

$$\mathbf{J} = \begin{pmatrix} 1+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\gamma_1 \end{pmatrix}. \tag{8}$$

$$|\gamma_{1,2}| \sim 0.02$$

Systematics

PSF:

$\int f(\vec{x}_l) g(\vec{x_o} - \vec{x}_l) d^2 x_l$

Noise:

$f(\vec{x}_o) + n(\vec{x}_o)$

Weight:

$w_l = w + C\gamma e$

Selection:

$m_l = m + C\gamma e$

Blending:

$f_1(x_l) + f_2(\vec{x}_l + \Delta\vec{x}_2)...$

Outline

- Analytically Construct a shear estimator for isolated galaxy.
- Calibration of bias from blending with realistic image simulation.

Shear Estimator

Shear Distortion

Distortion Operator (Refregier & Bacon 2003)

Taylor Expand for small shear:

$$\begin{aligned} f(\mathbf{J}\vec{x}) &= f(\vec{x}) + [\gamma_1(x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}) + \gamma_2(x\frac{\partial}{\partial y} + y\frac{\partial}{\partial x})]f(\vec{x}) \\ &= \hat{J}f(\vec{x}) \end{aligned} \tag{9}$$

Shear Distortion operator:

$$\hat{J} = \hat{I} + i\gamma_1(\hat{x}\hat{p}_x - \hat{y}\hat{p}_y) + i\gamma_2(\hat{x}\hat{p}_y + \hat{y}\hat{p}_x), \tag{10}$$

Position Operator and Momentum Operator:

$$\hat{x} = x \qquad \hat{y} = y, \tag{11}$$

$$\hat{p}_x = -i\frac{\partial}{\partial x} \qquad \hat{p}_y = -i\frac{\partial}{\partial y}. \tag{12}$$

Raising and Lowering Operator

The raising and lowering estimator for Quantum Harmonic Oscillator.

$$\hat{a}_x = \hat{x} + i\hat{p}_x \qquad \hat{a}_x^\dagger = \hat{x} - i\hat{p}_x \tag{13}$$

$$\hat{a}_y = \hat{x} + i\hat{p}_y \qquad \hat{a}_y^\dagger = \hat{x} - i\hat{p}_y \tag{14}$$

Distortion Operator can be written into raising and lowering operator

$$\begin{aligned}\hat{J} = \hat{I} - \frac{\gamma_1}{2}((\hat{a}_x^\dagger)^2 - (\hat{a}_y^\dagger)^2 - \hat{a}_x^2 + \hat{a}_y^2) \\ - \gamma_2(\hat{a}_x^\dagger \hat{a}_y^\dagger - \hat{a}_x \hat{a}_y)\end{aligned} \tag{15}$$

Quantum Harmonic Oscillator

- Response of egenstates to shear operator
- Decompose images with the egenstates

Polar shapelet basis vectors (Quantum Harmonic Oscillator in polar coordinates)

$$\chi_{nm}(r,\theta) = \frac{(-1)^{(n-|m|)/2}}{\sigma^{|m|+1}} \left\{ \frac{[(n-|m|)/2]!}{\pi[(n+|m|)/2]!} \right\}^{\frac{1}{2}} \times r^{|m|} L^{|m|}_{\frac{n-|m|}{2}} \left(\frac{r^2}{\sigma^2} \right) e^{-r^2/2\sigma^2} e^{-im\theta},$$

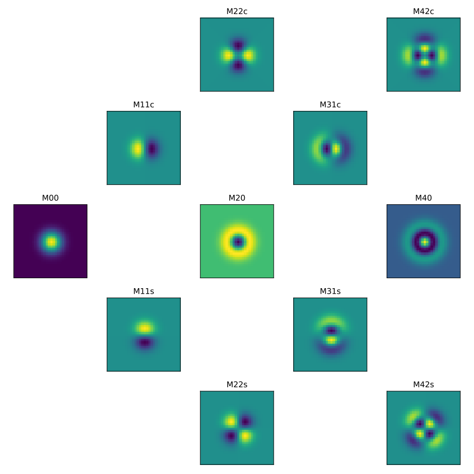
where $L^{|m|}_{\frac{n-|m|}{2}}$ is the Laguerre polynomials.

Project galaxy image (or galaxy's Fourier transform) onto the shapelet basis

$$M_{nm} = \int \chi^{\dagger}_{nm} f(r,\theta) r dr d\theta. \tag{16}$$

(Massey & Refregier 2005)

- ‘c’ refers to real part
- ‘s’ refers to imaginary part



Transformation of shapelet modes (M_{22}) under influence of shear:

$$\begin{aligned} \mathrm{M}_{22c} &= \bar{\mathrm{M}}_{22c} + \frac{\sqrt{2}}{2}\gamma_1(\bar{\mathrm{M}}_{00} - \bar{\mathrm{M}}_{40}) - \sqrt{3}\gamma_1\bar{\mathrm{M}}_{44c} - \sqrt{3}\gamma_2\bar{\mathrm{M}}_{44s}, \\ \mathrm{M}_{22s} &= \bar{\mathrm{M}}_{22s} + \frac{\sqrt{2}}{2}\gamma_2(\bar{\mathrm{M}}_{00} - \bar{\mathrm{M}}_{40}) + \sqrt{3}\gamma_2\bar{\mathrm{M}}_{44c} - \sqrt{3}\gamma_1\bar{\mathrm{M}}_{44s}. \end{aligned} \tag{17}$$

$$\begin{aligned} \langle \mathrm{M}_{22c} \rangle &= \frac{\sqrt{2}}{2}\gamma_1 \left\langle \bar{\mathrm{M}}_{00} - \bar{\mathrm{M}}_{40} \right\rangle, \\ \langle \mathrm{M}_{22s} \rangle &= \frac{\sqrt{2}}{2}\gamma_2 \left\langle \bar{\mathrm{M}}_{00} - \bar{\mathrm{M}}_{40} \right\rangle. \end{aligned} \tag{18}$$

(Massey & Refregier 2005)

Good Properties:

- Shear correlates a finite number of shapelets modes;
- Shapelet basis are orthogonal;

PSF Smearing

$$f_o(\vec{x}_o) = \int f(\vec{x}_l) g(\vec{x}_o - \vec{x}_l) d^2 x_l$$

Refregier & Bacon 2003

- PSF revision in Shapelet space.
- However, PSF correlates a infinite number of modes.

Massey & Refregier 2005

- Convolve Shapelets basis with PSF;
- Fit smeared galaxy image with the convolved basis.
- However, the convolved basis no longer orthogonal.

Fourier Power Function

The Fourier power function of the galaxy is defined as

$$\tilde{f}_o(\vec{k}) = \int f_o(\vec{x}_o) e^{-i\vec{k}\vec{x}_o} d^2x_o,$$
$$\tilde{F}_o(\vec{k}) = |\tilde{f}_o(\vec{k})|^2.$$

PSF Deconvolution

$$\tilde{F}(\vec{k}) = \frac{\tilde{F}_o(\vec{k})}{\tilde{G}(\vec{k})}$$

Well-defined Centroid

$$\tilde{F}_o(\vec{k}) = \tilde{F}_o(-\vec{k})$$

Project deconvolved Galaxy Fourier Power onto shapelet basis

$$M_{nm} = \int \chi^{\dagger}_{nm} \frac{\tilde{F}_o - \tilde{N}}{\tilde{G}} d^2k. \tag{19}$$

Normalization (Weighting)

Unweighted Shear Estimator

$$
\begin{aligned}
\langle M_{22c} \rangle &= \frac{\sqrt{2}}{2}\gamma_1 \left\langle \bar{M}_{00} - \bar{M}_{40} \right\rangle , \\
\langle M_{22s} \rangle &= \frac{\sqrt{2}}{2}\gamma_2 \left\langle \bar{M}_{00} - \bar{M}_{40} \right\rangle .
\end{aligned} \tag{20}
$$

Here I give out one construction of shear estimator:

$$
e_1 = \frac{M_{22c}}{M_{00} + C}, \qquad e_2 = \frac{M_{22s}}{M_{00} + C}.
$$

$$
\begin{aligned}
R_{1,2} &= \frac{\sqrt{2}}{2} \frac{M_{00} - M_{40}}{M_{00} + C} + \sqrt{2} e_{1,2}^2 , \\
\gamma_{1,2} &= \frac{\langle e_{1,2} \rangle}{\langle R_{1,2} \rangle}.
\end{aligned} \tag{21}
$$

(Li et. al 2018)

Noise Bias

$\mathcal{E}_{00}$ denotes the noise residuals after subtracting the power function of noise.

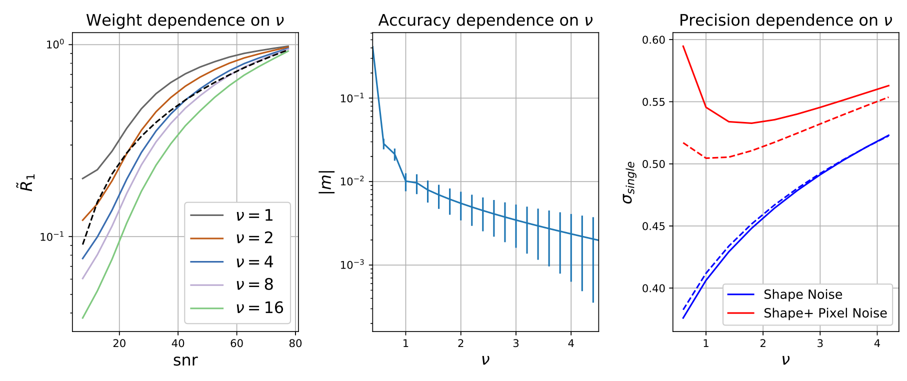
$$\left\langle \frac{\mathrm{M}_{22c} + \mathcal{E}_{22c}}{\mathrm{M}_{00} + \mathrm{C} + \mathcal{E}_{00}} \right\rangle \neq \left\langle \frac{\mathrm{M}_{22c}}{\mathrm{M}_{00} + \mathrm{C}} \right\rangle \tag{22}$$

Taylor expansion

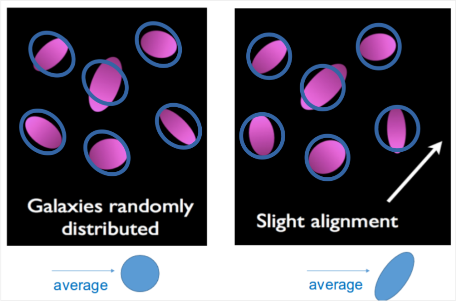
$$\left\langle \frac{\mathrm{M}_{22c} + \mathcal{E}_{22c}}{\mathrm{M}_{00} + \mathrm{C} + \mathcal{E}_{00}} \right\rangle = \left\langle \frac{\mathrm{M}_{22c}}{\mathrm{M}_{00} + \mathrm{C}} (1 + (\frac{\mathcal{E}_{00}}{\mathrm{M}_{00} + \mathrm{C}})^2) \right\rangle \tag{23}$$

Requires $\frac{\mathcal{E}_{00}}{\mathrm{M}_{00}+\mathrm{C}} \ll 1$ (Li et. al 2018)

Noise bias and weighting



$\nu = \frac{C}{\Delta}$, which is the normalized weighting parameter. Set $\nu = 4$.



The FPFS flux is defined as

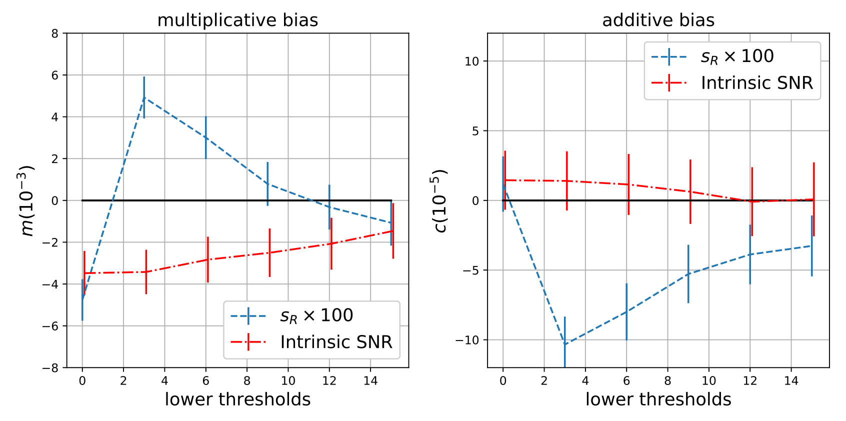
$$\mathrm{s} = \frac{\mathrm{M}_{00}}{\mathrm{M}_{00} + \mathrm{C}},$$

and the transformation of FPFS flux under the influence of shear

$$\mathrm{s} = \bar{\mathrm{s}} + \sqrt{2}\gamma_1 \mathrm{e}_1(1-\mathrm{s}) + \sqrt{2}\gamma_2 \mathrm{e}_2(1-\mathrm{s}).$$

BDK simulations (10^9 galaxies)

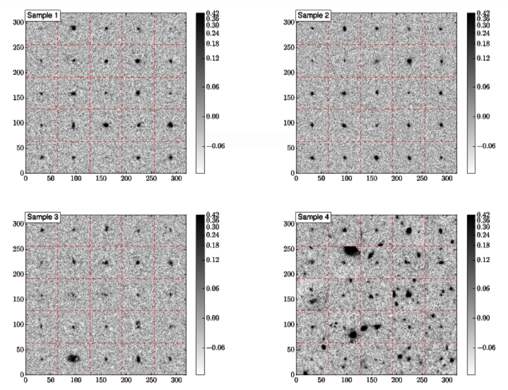
Bulge+Disk+Knots (BDK) simulation of Sheldon & Huff (2017).



For Isolated galaxies multiplicative bias can be controlled below 0.6% (MetaCal: 0.1%)

Realistic Image Simulation

Input Galaxies



Input Shear

1. Same shear/sub-sample
2. 800 sub-samples

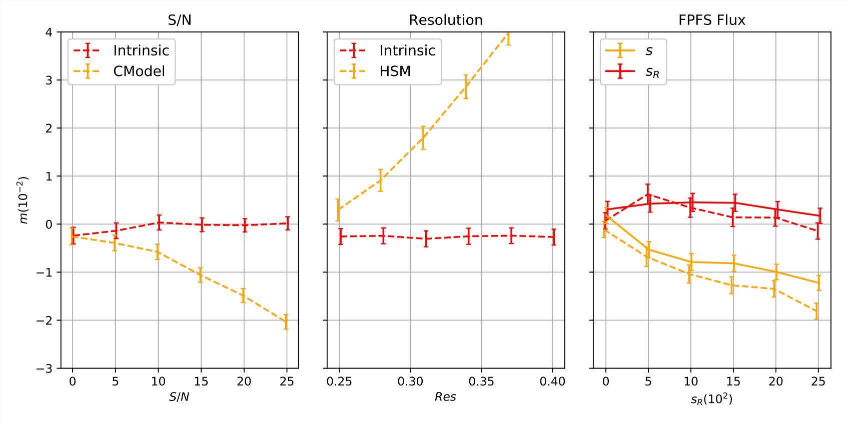
Input Noises

1. Correlation is estimated from blank pixels (average).
2. variance are sampled from HSC co-adds.

Input PSFs

1. Same PSF/sub-sample
2. PSF images are sampled from HSC co-adds.

Sample 1 and 2



dashed lines: Sample 1;
solid lines: Sample 2.

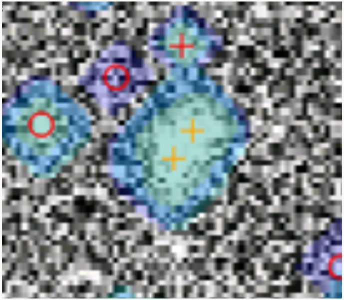
Sample 3 and 4

Deblend (D) v.s. Nodeblend (ND)

sample & setup	$m_1(10^{-2})$	$c_1(10^{-4})$	$m_2(10^{-2})$	$c_2(10^{-4})$
S3ND	-0.25 ± 0.22	0.75 ± 0.56	0.03 ± 0.23	-0.71 ± 0.59
S3D	-5.71 ± 0.24	3.33 ± 0.60	-5.59 ± 0.24	-1.06 ± 0.60
S4ND	-1.68 ± 0.27	0.24 ± 0.71	-1.11 ± 0.23	0.19 ± 0.57
S4D	-5.83 ± 0.41	1.27 ± 1.06	-5.59 ± 0.30	-0.71 ± 0.75

Note:

- Using SDSS Deblender
- '+' children in same footprint.
- 'o' sources not in the same footprint.



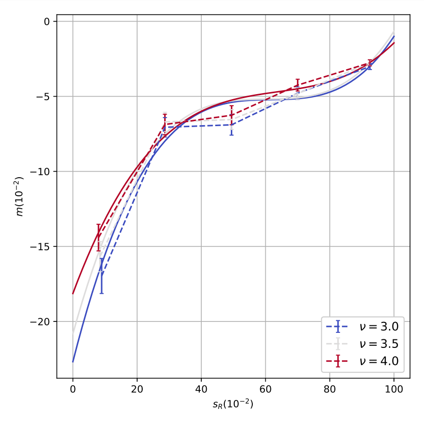
Realistic samples containing blended galaxies.

Calibrated Estimator

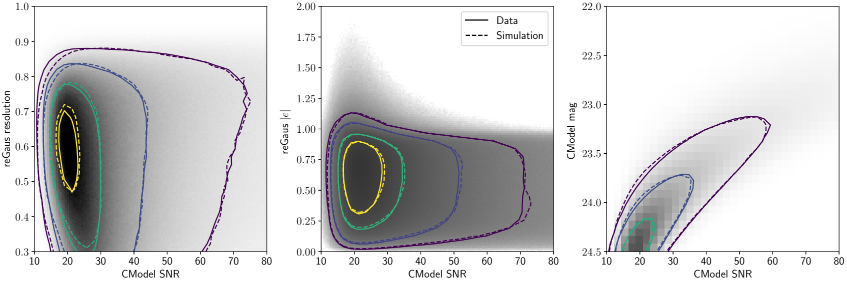
Model the multiplicative bias as a function of FPFS flux (s) and then revise the shear estimator.

$$\hat{g}_{1,2} = \frac{\langle e_{1,2} \rangle}{\langle (1 + m(s)) R_{1,2} \rangle}.$$

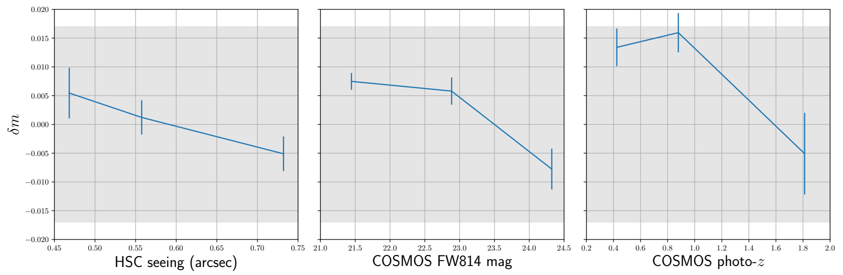
The calibration factor (m) is determined by sample 4 of the GREAT3 HSC-like simulations (Mandelbaum et al. 2018).



2-D Histograms

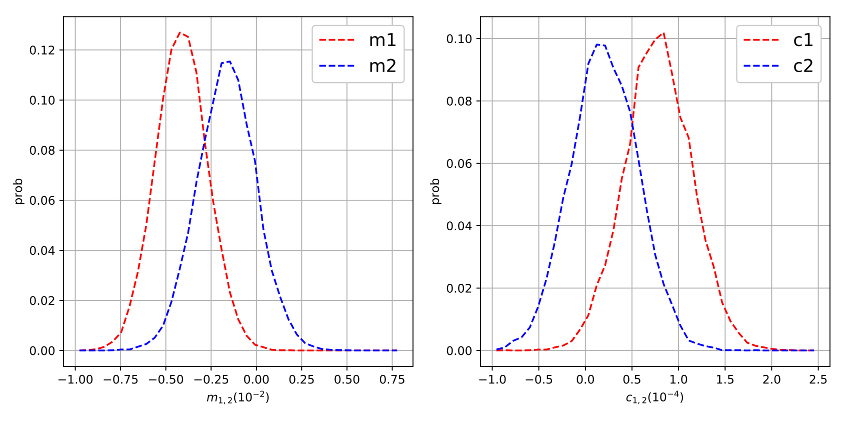


Bias of the calibrated estimator for Subsamples



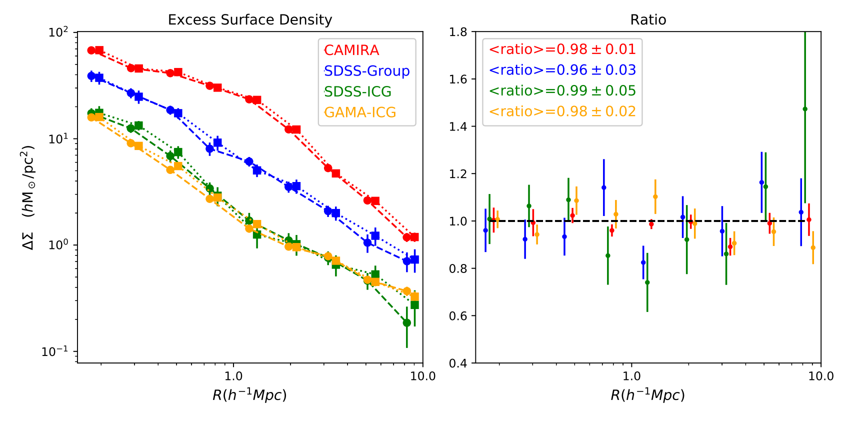
(Li et. al 2020)

Bias of the calibrated estimator for Bootstrapped Subsamples



(Li et. al 2020)

Results



(Li et. al 2020)

Thanks!