Fourier Power Function Shapelets (FPFS) Shear Estimator

Application to Hyper Suprime-Cam Survey

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March 10, 2020

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Method

Shear Measruement

Image Distortion

Denote the surface brightness distribution of a galaxy as $f(\boldsymbol{x})$

$$\begin{aligned} \boldsymbol{x}_l &= \mathbf{J}\boldsymbol{x}_s, \\ f_l(\boldsymbol{x}_l) &= f_s(\boldsymbol{x}_s), \end{aligned} \tag{1}$$

where the Jacobi matrix ($\mathbf{J}$) is

$$\mathbf{J} = \begin{pmatrix} 1+\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1+\kappa-\gamma_1 \end{pmatrix}. \tag{2}$$

Systematics

PSF:

$\int f(\boldsymbol{x}_l)p(\boldsymbol{x}-\boldsymbol{x}_l)d^2x_l$

Weight:

$w_l = w_s + C\sum_i g_i e_i$

Noise:

$f(\boldsymbol{x}_l) + n(\boldsymbol{x}_l)$

Selection:

$m_l = m_s + C\sum_i g_i e_i$

Blending:

$f_1(x_l) + f_2(\boldsymbol{x}_l + \Delta\boldsymbol{x}_2)...$

Fourier Power Function Shapelets Shear Estimator

Fourier Power Function

- PSF
- Centroid
- PF of Noise

Zhang & Komatsu 2011,
Zhang, Luo & Foucaud 2015.

HSC deblender+Calibration
Blending bias

Shapelets

- Noise Bias
- Weight Bias
- Selection Bias

Refregier & Bacon 2003,
Massey & Refregier 2005.

Polar Shapelets

Polar shapelet basis vectors (Quantum Harmonic Oscillator in polar coordinates)

$$\chi_{nm}(r,\theta) = \frac{(-1)^{(n-|m|)/2}}{\sigma^{|m|+1}} \left\{ \frac{[(n-|m|)/2]!}{\pi[(n+|m|)/2]!} \right\}^{\frac{1}{2}} \times r^{|m|} L^{|m|}_{\frac{n-|m|}{2}} \left(\frac{r^2}{\sigma^2} \right) e^{-r^2/2\sigma^2} e^{-im\theta},$$

where $L^{|m|}_{\frac{n-|m|}{2}}$ is the Laguerre polynomials.
Projecting galaxy image onto shapelet basis

$$M_{nm} = \int \chi^*_{nm} f(r,\theta) r dr d\theta. \tag{3}$$

Transform of shapelet modes

Transformation of shapelet modes (M_{22}) under influence of shear:

$$
\begin{aligned}
M_{22c} &= \bar{M}_{22c} + \frac{\sqrt{2}}{2}\gamma_1(\bar{M}_{00} - \bar{M}_{40}) - \sqrt{3}\gamma_1\bar{M}_{44c} - \sqrt{3}\gamma_2\bar{M}_{44s}, \\
M_{22s} &= \bar{M}_{22s} + \frac{\sqrt{2}}{2}\gamma_2(\bar{M}_{00} - \bar{M}_{40}) + \sqrt{3}\gamma_2\bar{M}_{44c} - \sqrt{3}\gamma_1\bar{M}_{44s}.
\end{aligned}
\tag{4}
$$

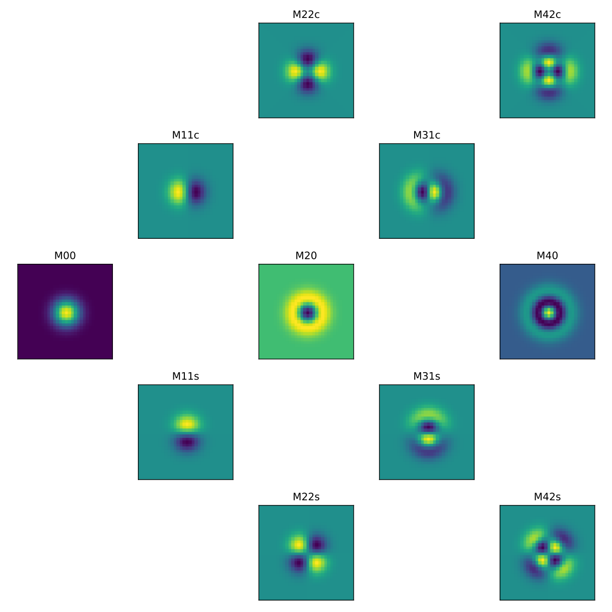
$$
\begin{aligned}
\langle M_{22c}\rangle &= \frac{\sqrt{2}}{2}\gamma_1\left\langle \bar{M}_{00} - \bar{M}_{40}\right\rangle, \\
\langle M_{22s}\rangle &= \frac{\sqrt{2}}{2}\gamma_2\left\langle \bar{M}_{00} - \bar{M}_{40}\right\rangle.
\end{aligned}
\tag{5}
$$

Refregier & Bacon 2003,

Massey & Refregier 2005.

Shear correlates finite number of shapelets modes; while PSF correlates infinite number of modes.

Shapelet basis vectors



Fourier Power Function

Fourier Transform

The Fourier power function of the galaxy is defined as

$$\tilde{f}_o(\boldsymbol{k}) = \int f_o(\boldsymbol{x}_o) e^{-i\boldsymbol{k}\boldsymbol{x}_o} d^2 x_o,$$
$$\tilde{F}_o(\boldsymbol{k}) = |\tilde{f}_o(\boldsymbol{k})|^2.$$

Top-hat aperture

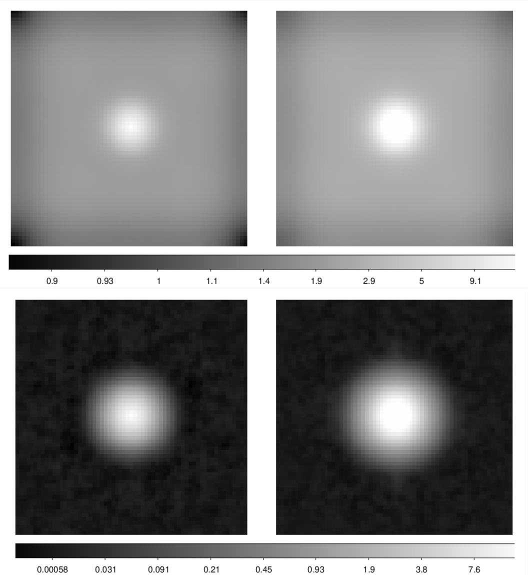
Before Fourier Transform

$$T(\boldsymbol{x}) = \begin{cases} 1, & |\boldsymbol{x} - \boldsymbol{x}_c| < r_{\mathrm{cut}} \\ 0, & |\boldsymbol{x} - \boldsymbol{x}_c| \geq r_{\mathrm{cut}} \end{cases}, \tag{6}$$

The aperture radius ratio $\alpha = r_{\mathrm{cut}}/r_g$

Model the noise power function and subtract it for each galaxy

$$\tilde{F}_o - \tilde{F}_n \qquad (7)$$



PSF Deconvolution

PSF Effect

PSF Effect is modelled as a convolution between lensed galaxy (f_l) and PSF (g).

$$f_o(\boldsymbol{x}_o) = \int d^2x f_l(\boldsymbol{x}_o - \boldsymbol{x}) g(\boldsymbol{x}) \tag{8}$$

The PSF effect can be removed by dividing the PSF Fourier power function ($\tilde{G}$) from the observed galaxy Fourier power function

$$\tilde{F}(\boldsymbol{k}) = \frac{\tilde{F}_o(\boldsymbol{k})}{\tilde{G}(\boldsymbol{k})}. \tag{9}$$

Project deconvolved FPF to Shapelets Space

Polar shapelet basis vectors

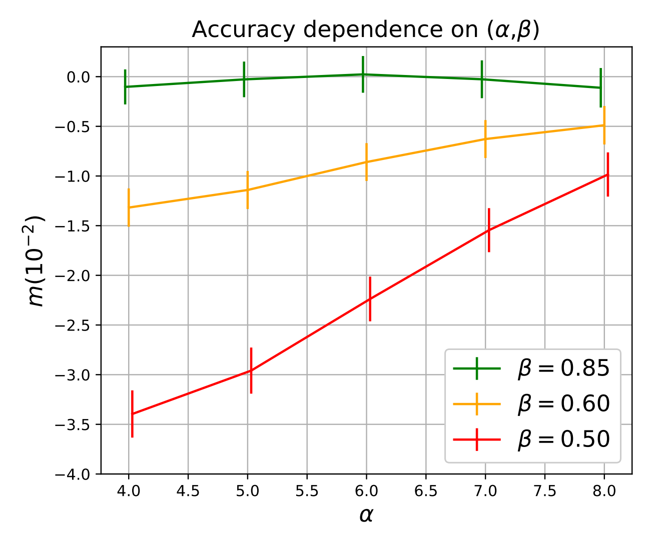
$$\chi_{nm}(r,\theta) = \frac{(-1)^{(n-|m|)/2}}{\sigma^{|m|+1}} \left\{ \frac{[(n-|m|)/2]!}{\pi[(n+|m|)/2]!} \right\}^{\frac{1}{2}} \times r^{|m|} L_{\frac{n-|m|}{2}}^{|m|} \left(\frac{r^2}{\sigma^2} \right) e^{-r^2/2\sigma^2} e^{-im\theta},$$

Projecting the deconvolved Fourier Power function onto shapelet basis

$$M_{nm} = \int \chi^*_{nm} \tilde{F}(r,\theta) r dr d\theta. \tag{10}$$

Denote $\beta = \sigma / r_{P\tilde{S}F}$ ($\beta < 1$). Remind: $\alpha = r_{\mathrm{cut}}/r_g$
α determines the boundary of galaxies and β determines the smoothing scale.

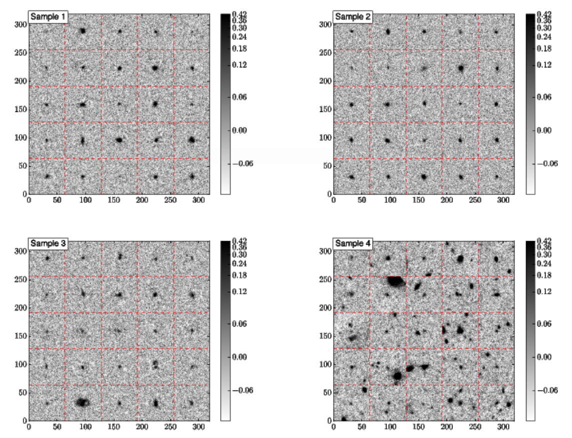
Boundary (α) and Smoothing (β)



Li et al. 2018

Image Simulations

Great3 HSC-like simulations



Mandelbaum et al. 2018

$$\hat{\gamma}_{1,2} = (1 + m_{1,2})\gamma_{1,2} + c_{1,2}, \tag{11}$$

Transform of shapelet modes

Transformation of shapelet modes (M_{22}) under influence of shear:

$$\begin{aligned} M_{22c} &= \bar{M}_{22c} - \frac{\sqrt{2}}{2}\gamma_1(\bar{M}_{00} - \bar{M}_{40}) + \sqrt{3}\gamma_1\bar{M}_{44c} + \sqrt{3}\gamma_2\bar{M}_{44s}, \\ M_{22s} &= \bar{M}_{22s} - \frac{\sqrt{2}}{2}\gamma_2(\bar{M}_{00} - \bar{M}_{40}) - \sqrt{3}\gamma_2\bar{M}_{44c} + \sqrt{3}\gamma_1\bar{M}_{44s}. \end{aligned} \tag{12}$$

$$\begin{aligned} \langle M_{22c}\rangle &= -\frac{\sqrt{2}}{2}\gamma_1\left\langle \bar{M}_{00} - \bar{M}_{40}\right\rangle, \\ \langle M_{22s}\rangle &= -\frac{\sqrt{2}}{2}\gamma_2\left\langle \bar{M}_{00} - \bar{M}_{40}\right\rangle. \end{aligned} \tag{13}$$

Only need 4 shapelet modes to infer shear (Zhang & Komatsu 2011).

Normalization (Weighting)

$$e_1 = \frac{M_{22c}}{M_{00} + C}, \qquad e_2 = \frac{M_{22s}}{M_{00} + C}.$$

$$R_{1,2} = \frac{\sqrt{2}}{2} \frac{M_{00} - M_{40}}{M_{00} + C} + \sqrt{2} e_{1,2}^2,$$

$$e_1 = \bar{e}_1 - \gamma_1 \bar{R}_1 + \sqrt{3}\gamma_1 \frac{\bar{M}_{44c}}{\bar{M}_{00} + C} + \sqrt{3}\gamma_2 \frac{\bar{M}_{44s}}{\bar{M}_{00} + C},$$
$$e_2 = \bar{e}_2 - \gamma_2 \bar{R}_2 - \sqrt{3}\gamma_2 \frac{\bar{M}_{44c}}{\bar{M}_{00} + C} + \sqrt{3}\gamma_1 \frac{\bar{M}_{44s}}{\bar{M}_{00} + C}.$$

$$\gamma_{1,2} = -\frac{\langle e_{1,2} \rangle}{\langle R_{1,2} \rangle}. \tag{14}$$

Noise Bias

N_{00} denotes the noise residuals after subtracting the power function of noise.

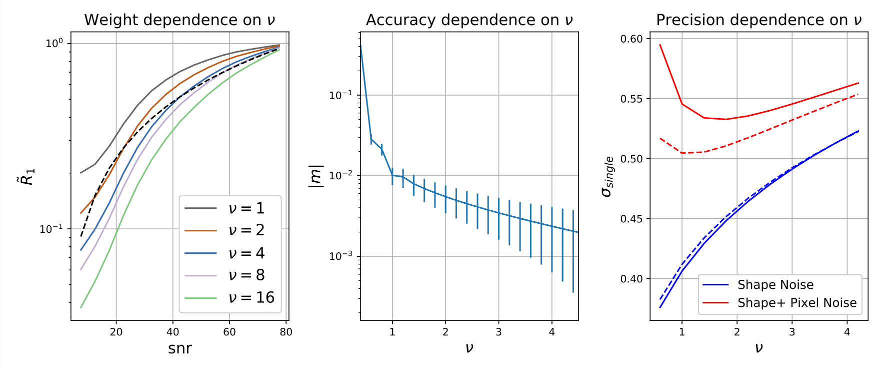
$$\left\langle \frac{M_{22c} + N_{22c}}{M_{00} + C + N_{00}} \right\rangle \neq \left\langle \frac{M_{22c}}{M_{00} + C} \right\rangle \tag{15}$$

Taylor expansion

$$\left\langle \frac{M_{22c} + N_{22c}}{M_{00} + C + N_{00}} \right\rangle = \left\langle \frac{M_{22c}}{M_{00} + C} (1 + (\frac{N_{00}}{M_{00} + C})^2) \right\rangle \tag{16}$$

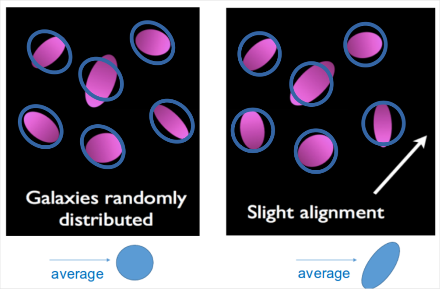
Roughly Speaking
By ensuring $C > 10|N_{00}|$, noise bias can be controlled below 1%.

Noise bias and weighting



$\nu = \frac{C}{\Delta}$, which is the normalized weighting parameter.

Selection bias



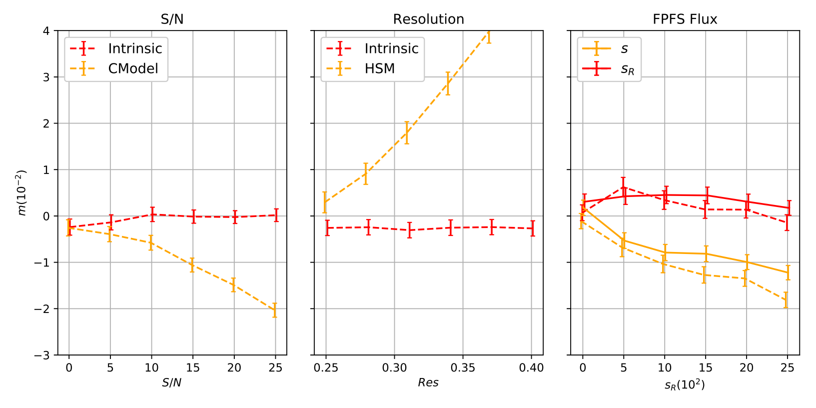
The FPFS flux is defined as

$$s = \frac{M_{00}}{M_{00} + C},$$

and the transformation of FPFS flux under the influence of shear

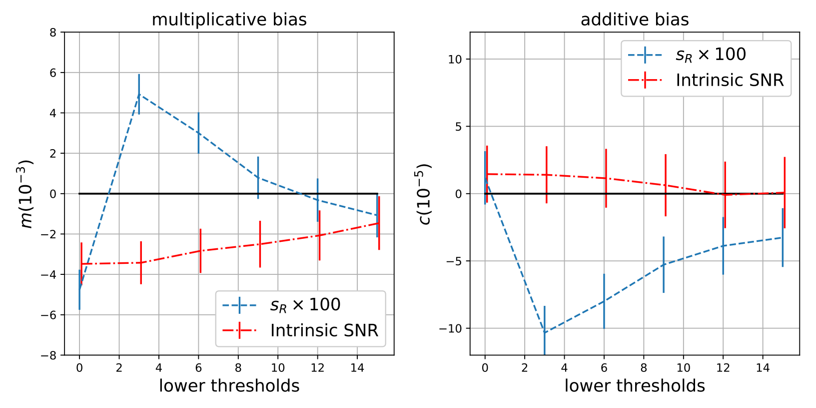
$$s = \bar{s} + \sqrt{2}\gamma_1 e_1 (1 - s) + \sqrt{2}\gamma_2 e_2 (1 - s).$$

HSC-like GREAT3 Simulations



BDK-like simulations

Similar to the BDK simulation of Sheldon & Huff (2017).



For Isolated galaxies multiplicative bias can be controlled below 0.6%

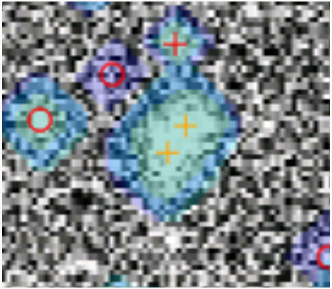
Blending

Deblend (D) v.s. Nodeblend (ND)

sample & setup	$m_1(10^{-2})$	$c_1(10^{-4})$	$m_2(10^{-2})$	$c_2(10^{-4})$
S3ND	-0.25 ± 0.22	0.75 ± 0.56	0.03 ± 0.23	-0.71 ± 0.59
S3D	-5.71 ± 0.24	3.33 ± 0.60	-5.59 ± 0.24	-1.06 ± 0.60
S4ND	-1.68 ± 0.27	0.24 ± 0.71	-1.11 ± 0.23	0.19 ± 0.57
S4D	-5.83 ± 0.41	1.27 ± 1.06	-5.59 ± 0.30	-0.71 ± 0.75

Note:

- Same shear on blended children.
- '+' children in same footprint.
- 'o' sources not in the same footprint.

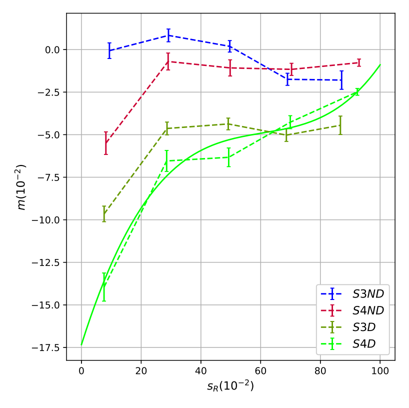


Realistic samples containing blended galaxies.

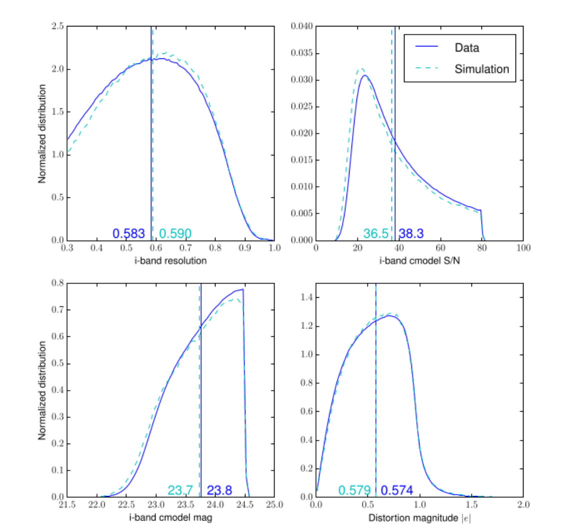
Calibrated Estimator
Model the multiplicative bias as a function of FPFS flux (s) and then revise the shear estimator.

$$\hat{g}_{1,2} = \frac{\langle e_{1,2} \rangle}{\langle (1 + m(s)) R_{1,2} \rangle}.$$

The calibration factor (m) is determined by sample 4 of the GREAT3 HSC-like simulations (Mandelbaum et al. 2018).

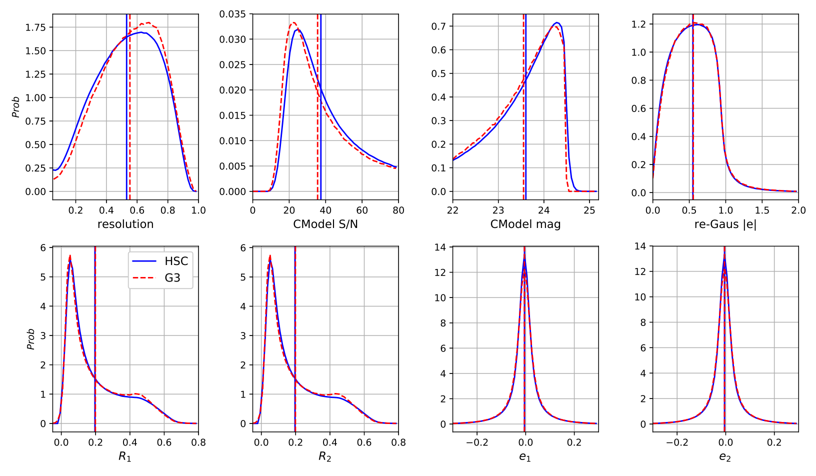


Simulation v.s. Observation



Mandelbaum et. al 2018

Simulation v.s. Observation



Li et. al 2018

Calibration uncertainty

Apply the calibration factor derived from the whole sample to subsamples which have different galaxy populations, observation conditions.

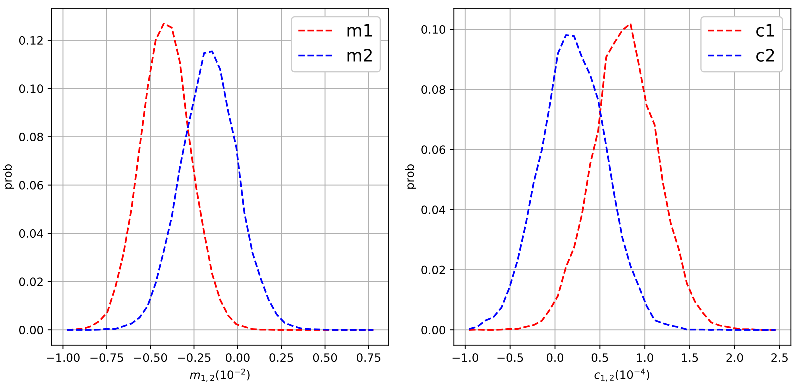
Subsamples with different PSF properties

sample	$\delta m(10^{-2})$	$\delta c(10^{-4})$
1st quartile	0.88 ± 0.56	-2.15 ± 1.48
2nd quartile	-0.36 ± 0.30	2.03 ± 0.73
3rd quartile	-0.49 ± 0.54	1.83 ± 1.35
4th quartile	-0.70 ± 0.86	-1.33 ± 2.23

Calibration uncertainty

Bootstrap 800 subsamples to simulate samples with different galaxy populations.

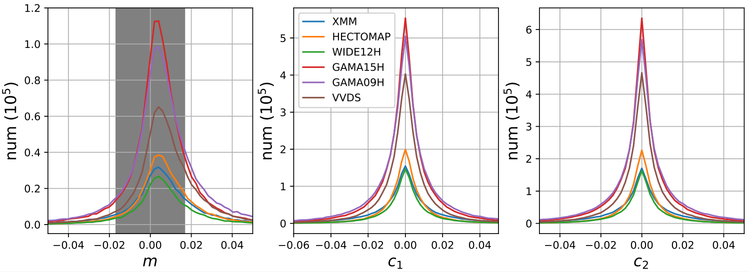
Each subsamples have 10^4 galaxies.

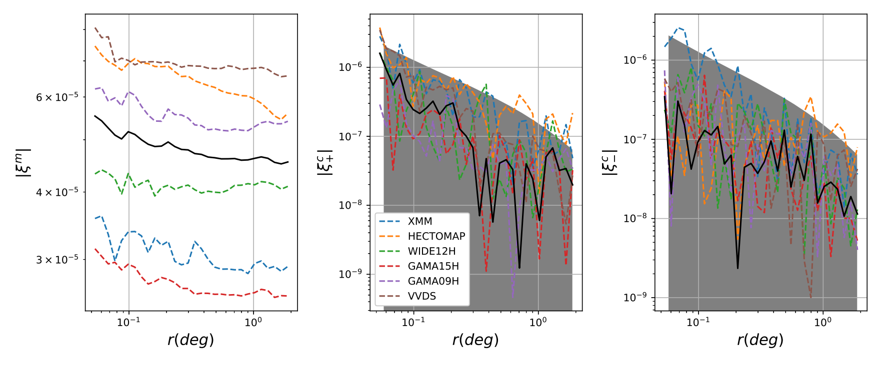


Application to Hyper Suprime-Cam (HSC)

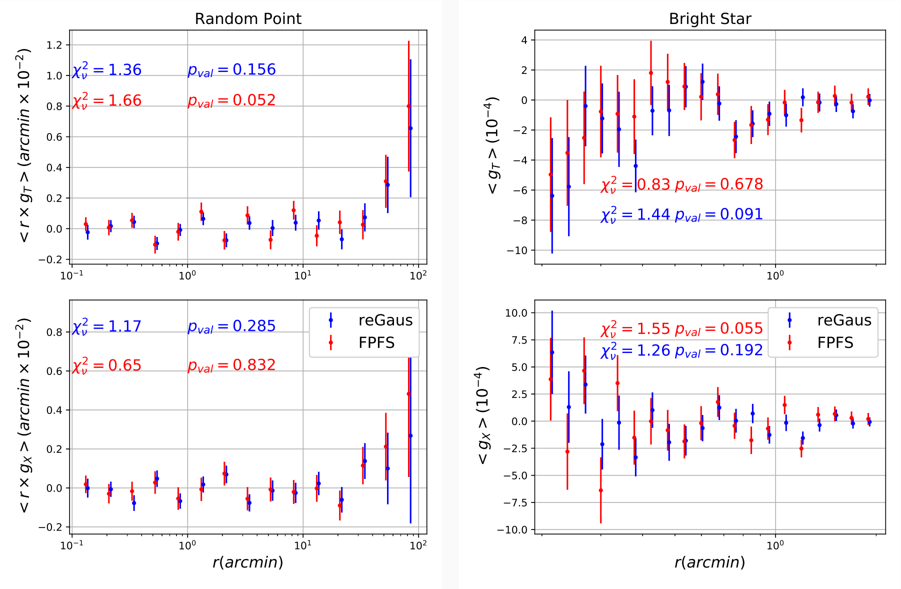
According to Lu et. al (2016),

- Choose bright stars, and the corresponding PSF;
- Simulate galaxies distorted by input shears;
- Convolve with star image and deconvolve with PSF image;
- estimate input shear and get biases m and $c_{1,2}$.

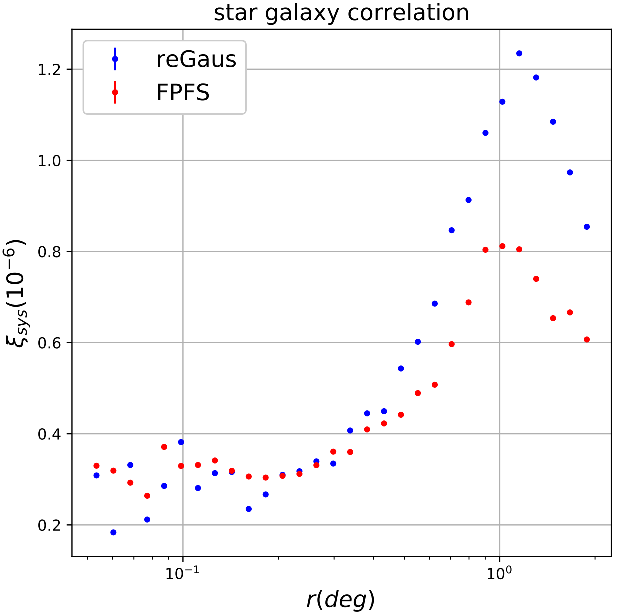




$|\xi^m|$
6×10^{-5}
4×10^{-5}
3×10^{-5}
10^{-1}
10^{0}
$r(deg)$
$|\xi^c_+|$
10^{-6}
10^{-7}
10^{-8}
10^{-9}
XMM
HECTOMAP
WIDE12H
GAMA15H
GAMA09H
VVDS
$|\xi^c_-|$

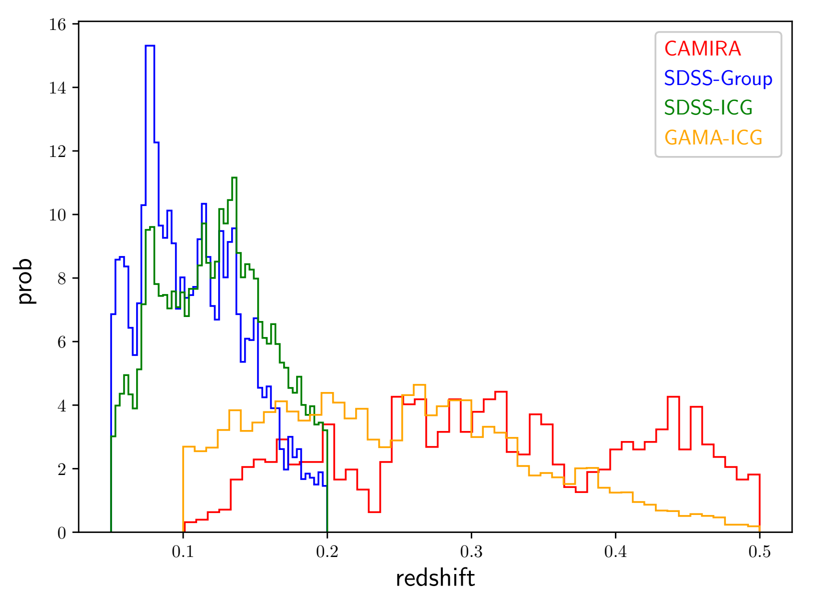


Random Point
Bright Star
$\chi^2_\nu = 1.36$
$p_{val} = 0.156$
$\chi^2_\nu = 1.66$
$p_{val} = 0.052$
$< r \times g_T > (arcmin \times 10^{-2})$
$\chi^2_\nu = 0.83$ $p_{val} = 0.678$
$\chi^2_\nu = 1.44$ $p_{val} = 0.091$
$< g_T > (10^{-4})$
$\chi^2_\nu = 1.17$
$p_{val} = 0.285$
$\chi^2_\nu = 0.65$
$p_{val} = 0.832$
$< r \times g_X > (arcmin \times 10^{-2})$
reGaus
FPFS
$\chi^2_\nu = 1.55$ $p_{val} = 0.055$
$\chi^2_\nu = 1.26$ $p_{val} = 0.192$
$< g_X > (10^{-4})$
$r(arcmin)$
$r(arcmin)$

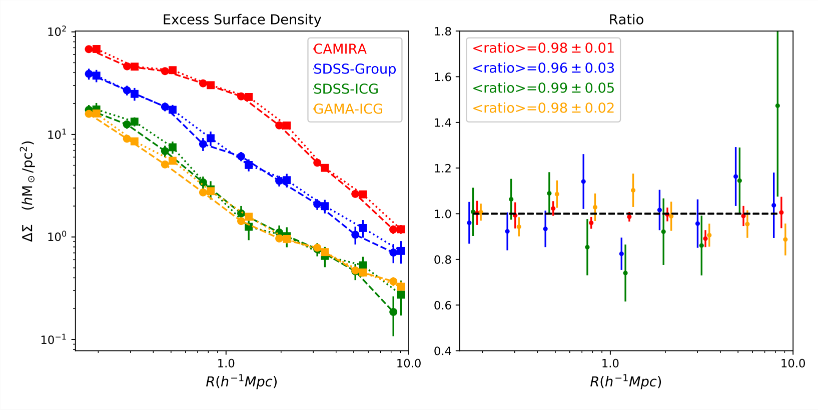

star galaxy correlation
reGaus
FPFS
1.2
1.0
0.8
0.6
0.4
0.2
$\xi_{sys}(10^{-6})$
10^{-1}
10^{0}
$r(deg)$

Galaxy-galaxy Lensing

Lens catalogs

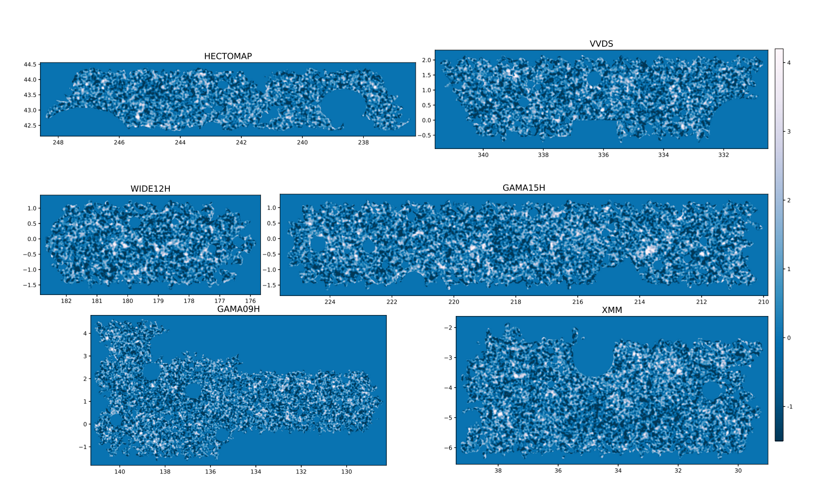


Results

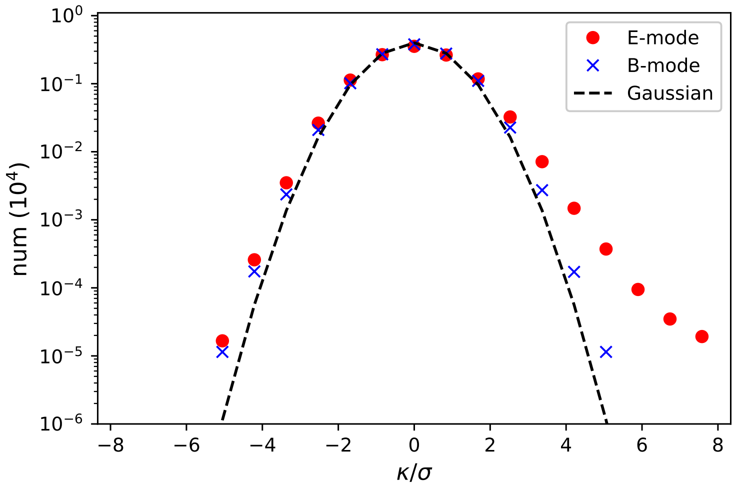

Excess Surface Density
CAMIRA
SDSS-Group
SDSS-ICG
GAMA-ICG
$\Delta\Sigma \quad (hM_\odot/pc^2)$
10^2
10^1
10^0
10^{-1}
1.0
10.0
$R(h^{-1}Mpc)$
Ratio
<ratio>=0.98 ± 0.01
<ratio>=0.96 ± 0.03
<ratio>=0.99 ± 0.05
<ratio>=0.98 ± 0.02
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
1.0
10.0
$R(h^{-1}Mpc)$

Mass map

mass map



PDF of E(B)-mode



Thanks