Fourier Power Function Shapelets (FPFS) Shear Estimator

Application to Hyper Suprime-Cam Survey

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HSC collaboration

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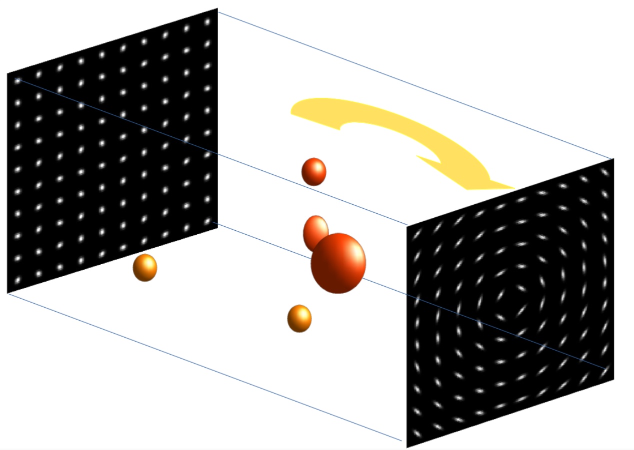
Hmm...

It is a technical report....might be boring.
Sorry in advance.

Weak Lensing

Weak Lensing



$$\underbrace{\kappa(\boldsymbol{\theta}, w)}_{\text{shape}} = \frac{3H_0^2\Omega_M}{2c^2}\int_0^w dw' \underbrace{\frac{w'(w-w')}{w\ a(w')}}_{\text{photo-}z} \underbrace{\delta(w'\boldsymbol{\theta}, w')}_{\text{density}}.$$

κ & $\gamma_{1,2}$

Image Distortion

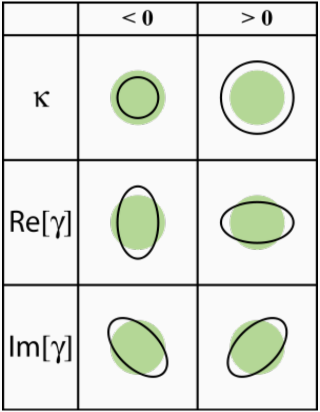
Denote the surface brightness distribution of a galaxy as $f(\boldsymbol{x})$

Jacobi Matrix

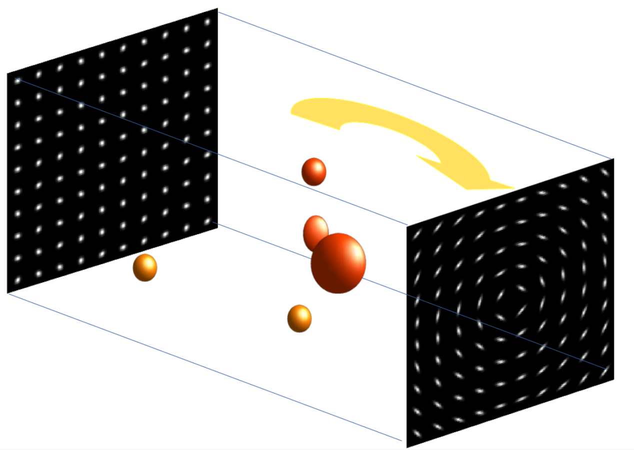
$$\begin{aligned} \boldsymbol{x}_l &= \mathbf{J}\boldsymbol{x}_s, \\ f_l(\boldsymbol{x}_l) &= f_s(\boldsymbol{x}_s), \end{aligned} \tag{1}$$

where the Jacobi matrix ($\mathbf{J}$) is

$$\mathbf{J} = \begin{pmatrix} 1+\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1+\kappa-\gamma_1 \end{pmatrix}. \tag{2}$$



Weak Lensing

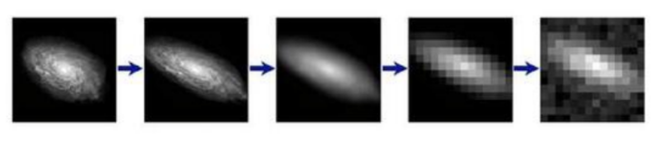


$\langle \bar{e}_{1,2} \rangle = 0,$ $\langle e_{1,2} \rangle \rightarrow \gamma_{1,2} \rightarrow \kappa$

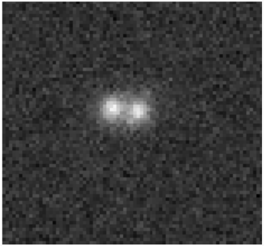
Systematics

$\gamma_{1,2} \sim 0.02$

Systematics in shear measurement

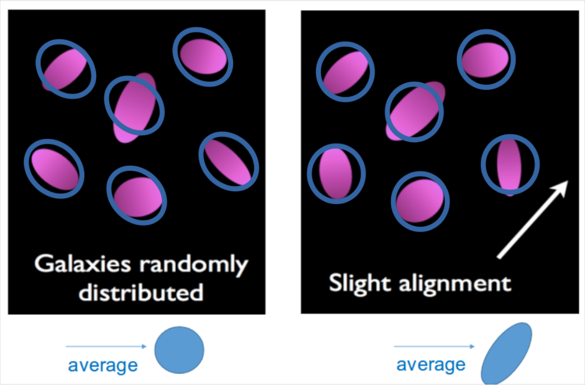


Blending (detected) in deep ground-based surveys

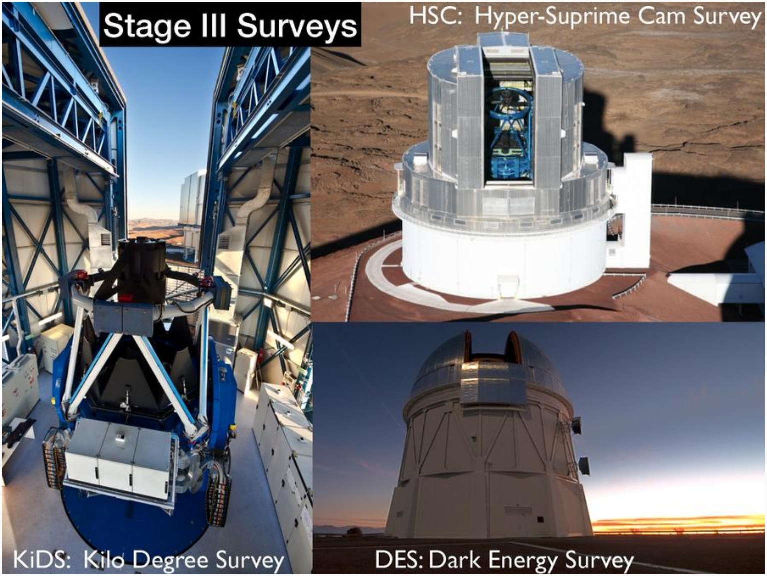


Systematics

selection bias



On-going surveys



Fourier Power Function Shapelets (FPFS) Shear Estimator

Fourier Power Function Shapelets Shear Estimator

Fourier Power Function

- PSF
- Centroid

Zhang & Komatsu 2011,
Zhang, Luo & Foucaud 2015.

HSC deblender+Calibration
Blending bias

Shapelets

- Ellipticity Definition
- Selection Bias

Refregier & Bacon 2003,
Massey & Refregier 2005.

Fourier Power Function

Fourier Transform

The Fourier power function of the galaxy is defined as

$$\tilde{f}_o(\boldsymbol{k}) = \int f_o(\boldsymbol{x}_o) e^{-i\boldsymbol{k}\boldsymbol{x}_o} d^2 x_o,$$
$$\tilde{F}_o(\boldsymbol{k}) = |\tilde{f}_o(\boldsymbol{k})|^2.$$

Top-hat aperture

Before Fourier Transform

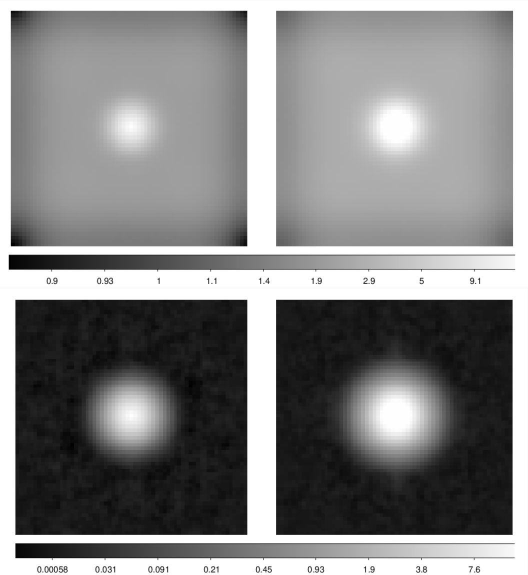
$$T(\boldsymbol{x}) = \begin{cases} 1, & |\boldsymbol{x} - \boldsymbol{x}_c| < r_{\rm cut} \\ 0, & |\boldsymbol{x} - \boldsymbol{x}_c| \geq r_{\rm cut} \end{cases}, \tag{3}$$

The aperture radius ratio $\alpha = r_{\rm cut}/r_g$

Noise Power Function

Model the noise power function and subtract it for each galaxy

$$\tilde{F}_o - \tilde{F}_n \qquad (4)$$



PSF Effect

PSF Effect is regarded as a convolution between lensed galaxy (f_l) and PSF (g).

$$f_o(\boldsymbol{x}_o) = \int d^2x f_l(\boldsymbol{x}_o - \boldsymbol{x})g(\boldsymbol{x}) \tag{5}$$

The PSF effect can be removed by dividing the PSF Fourier power function ($\tilde{G}$) from the observed galaxy Fourier power function

$$\tilde{F}(\boldsymbol{k}) = \frac{\tilde{F}_o(\boldsymbol{k})}{\tilde{G}(\boldsymbol{k})}. \tag{6}$$

Polar Shapelets

Polar shapelet basis vectors (Quantum Harmonic Oscillator in polar coordinates)

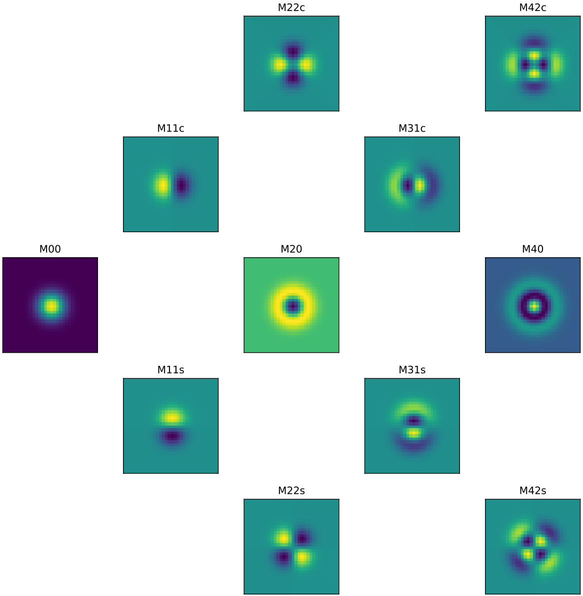
$$\chi_{nm}(r,\theta) = \frac{(-1)^{(n-|m|)/2}}{\sigma^{|m|+1}} \left\{ \frac{[(n-|m|)/2]!}{\pi[(n+|m|)/2]!} \right\}^{\frac{1}{2}} \times r^{|m|} L^{|m|}_{\frac{n-|m|}{2}} \left(\frac{r^2}{\sigma^2} \right) e^{-r^2/2\sigma^2} e^{-im\theta},$$

where $L^{|m|}_{\frac{n-|m|}{2}}$ is the Laguerre polynomials.
Projecting Fourier Power function onto shapelet basis

$$M_{nm} = \int \chi^*_{nm} \tilde{F}(r,\theta) r dr d\theta. \tag{7}$$

Denote $\beta = \sigma / r_{P\tilde{S}F}$ and $\beta < 1$.
α defines the boundary of galaxies and β defines the smoothing scale.

Shapelet basis vectors



Transform of shapelet modes

Transformation of shapelet modes (M_{22}) under influence of shear:

$$\begin{aligned} M_{22c} &= \bar{M}_{22c} - \frac{\sqrt{2}}{2}\gamma_1(\bar{M}_{00} - \bar{M}_{40}) + \sqrt{3}\gamma_1\bar{M}_{44c} + \sqrt{3}\gamma_2\bar{M}_{44s}, \\ M_{22s} &= \bar{M}_{22s} - \frac{\sqrt{2}}{2}\gamma_2(\bar{M}_{00} - \bar{M}_{40}) - \sqrt{3}\gamma_2\bar{M}_{44c} + \sqrt{3}\gamma_1\bar{M}_{44s}. \end{aligned} \tag{8}$$

$$\begin{aligned} \langle M_{22c} \rangle &= -\frac{\sqrt{2}}{2}\gamma_1 \left\langle \bar{M}_{00} - \bar{M}_{40} \right\rangle, \\ \langle M_{22s} \rangle &= -\frac{\sqrt{2}}{2}\gamma_2 \left\langle \bar{M}_{00} - \bar{M}_{40} \right\rangle. \end{aligned} \tag{9}$$

Only need 4 shapelet modes to infer shear (Zhang & Komatsu 2011).

Normalization

$$e_1 = \frac{M_{22c}}{M_{00} + C}, \qquad e_2 = \frac{M_{22s}}{M_{00} + C}.$$

$$R_i = \frac{\sqrt{2}}{2} \frac{M_{00} - M_{40}}{M_{00} + C} + \sqrt{2} e_i^2,$$

the transformation of the FPFS ellipticity under the influence of shear is given as follows

$$\begin{aligned}
e_1 &= \bar{e}_1 - \gamma_1 \bar{R}_1 + \sqrt{3}\gamma_1 \frac{\bar{M}_{44c}}{\bar{M}_{00} + C} + \sqrt{3}\gamma_2 \frac{\bar{M}_{44s}}{\bar{M}_{00} + C}, \\
e_2 &= \bar{e}_2 - \gamma_2 \bar{R}_2 - \sqrt{3}\gamma_2 \frac{\bar{M}_{44c}}{\bar{M}_{00} + C} + \sqrt{3}\gamma_1 \frac{\bar{M}_{44s}}{\bar{M}_{00} + C}.
\end{aligned}$$

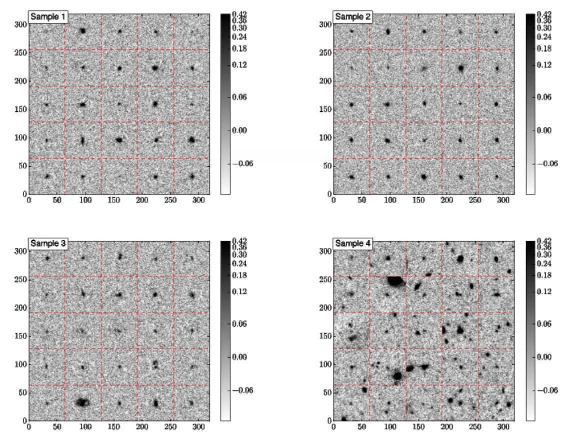
Shear Estimator

$$
\begin{aligned}
e_1 &= \bar{e}_1 - \gamma_1 \bar{R}_1 + \sqrt{3}\gamma_1 \frac{\bar{M}_{44c}}{\bar{M}_{00} + C} + \sqrt{3}\gamma_2 \frac{\bar{M}_{44s}}{\bar{M}_{00} + C}, \\
e_2 &= \bar{e}_2 - \gamma_2 \bar{R}_2 - \sqrt{3}\gamma_2 \frac{\bar{M}_{44c}}{\bar{M}_{00} + C} + \sqrt{3}\gamma_1 \frac{\bar{M}_{44s}}{\bar{M}_{00} + C}.
\end{aligned}
$$

With unbiased selection, the shear estimator is

$$
\gamma_{1,2} = -\frac{\langle e_{1,2} \rangle}{\langle R_{1,2} \rangle}. \tag{10}
$$

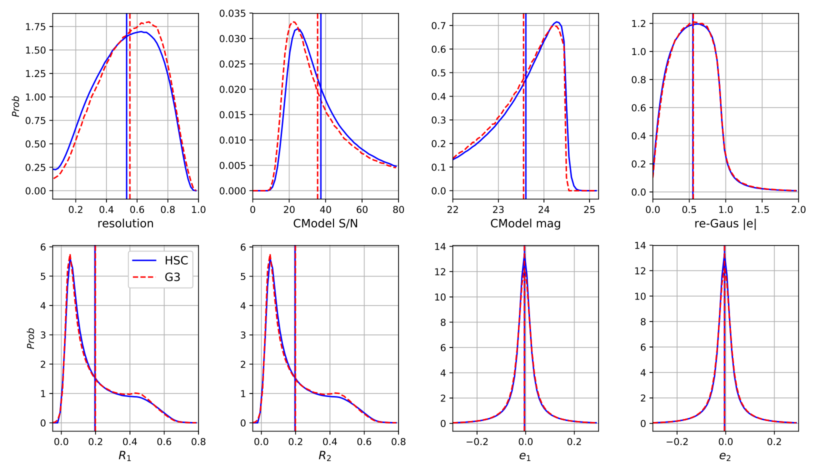
Great3 HSC-like simulations



$$\hat{\gamma}_{1,2} = (1 + m_{1,2})\gamma_{1,2} + c_{1,2}, \qquad (11)$$

where $m_{1,2}$ is termed multiplicative bias, $c_{1,2}$ is termed additive bias.

Simulation v.s. Observation



Noise bias

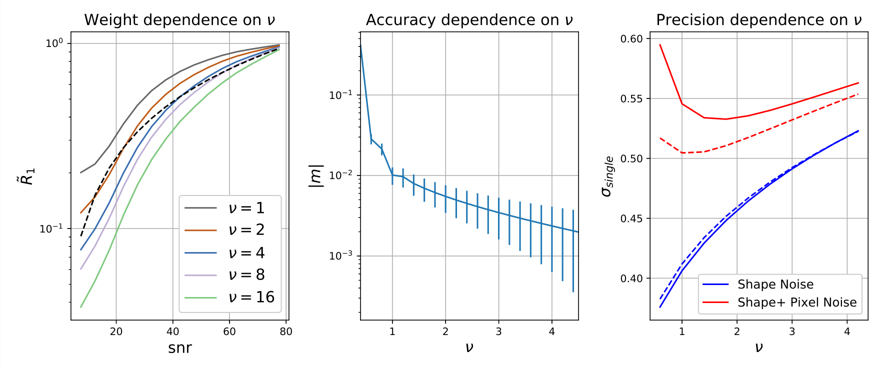
$$\left\langle \frac{M_{22c} + N_{22c}}{M_{00} + C + N_{00}} \right\rangle \neq \left\langle \frac{M_{22c}}{M_{00} + C} \right\rangle \tag{12}$$

Taylor expansion

$$\left\langle \frac{M_{22c} + N_{22c}}{M_{00} + C + N_{00}} \right\rangle = \left\langle \frac{M_{22c}}{M_{00} + C} (1 + (\frac{N_{00}}{M_{00} + C})^2) \right\rangle \tag{13}$$

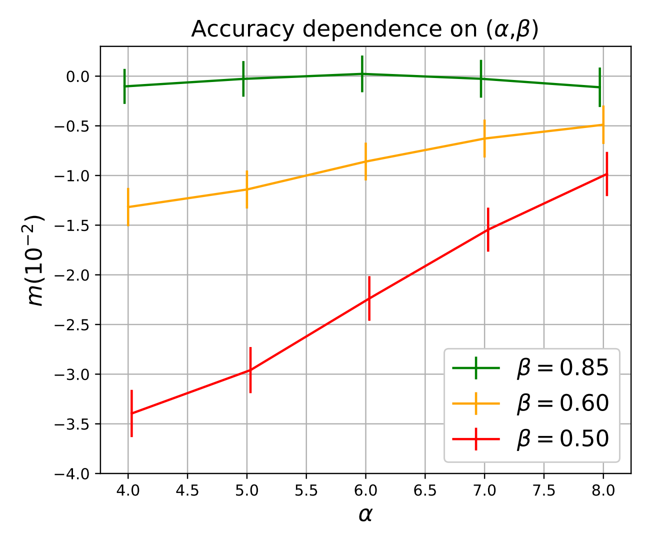
In order to limit noise bias below one percent, it is necessary to adjust $C > 10\sigma_N$.

Noise bias and weight

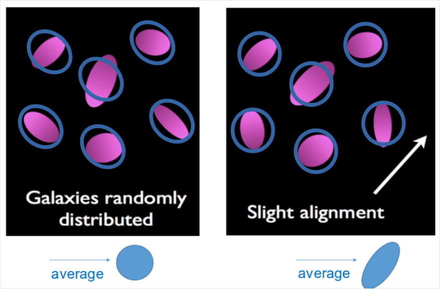


$\nu = \frac{C}{\Delta}$, which is the normalized weighting parameter.

Boundary (α) and Smoothing (β)



Selection bias



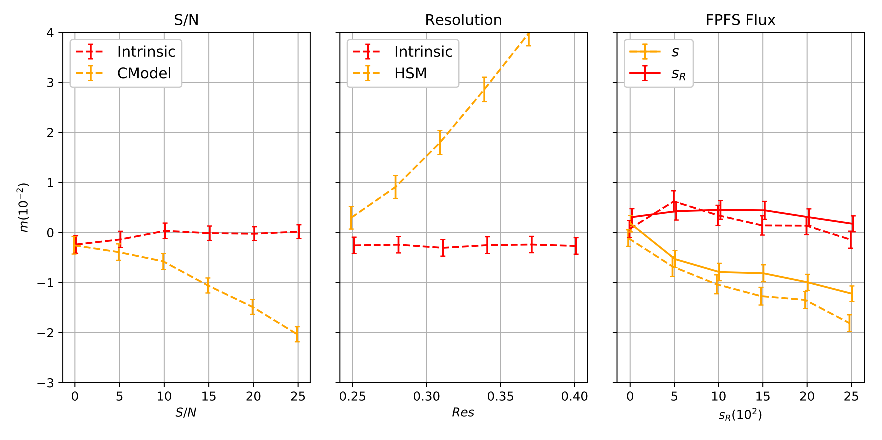
The FPFS flux is defined as

$$s = \frac{M_{00}}{M_{00} + C},$$

and the transformation of FPFS flux under the influence of shear

$$s = \bar{s} + \sqrt{2}\gamma_1 e_1(1 - s) + \sqrt{2}\gamma_2 e_2(1 - s).$$

Selection bias



Blending

Deblend or not

sample & setup	$m_1(10^{-2})$	$c_1(10^{-4})$	$m_2(10^{-2})$	$c_2(10^{-4})$
S3ND	-0.25 ± 0.22	0.75 ± 0.56	0.03 ± 0.23	-0.71 ± 0.59
S3D	-5.71 ± 0.24	3.33 ± 0.60	-5.59 ± 0.24	-1.06 ± 0.60
S4ND	-1.68 ± 0.27	0.24 ± 0.71	-1.11 ± 0.23	0.19 ± 0.57
S4D	-5.83 ± 0.41	1.27 ± 1.06	-5.59 ± 0.30	-0.71 ± 0.75

Note
For these simulations, blended galaxies are influenced by the same share.

Blending

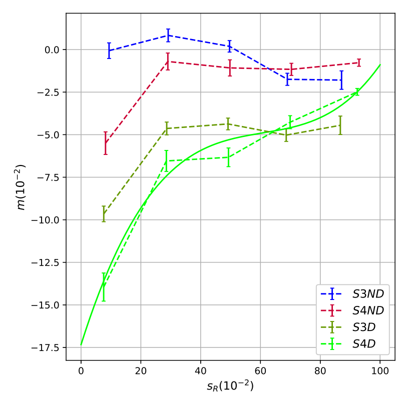
Realistic samples containing blended galaxies.

Calibrated Estimator

Model the multiplicative bias as a function of FPFS flux (s) and then revise the shear estimator.

$$\hat{g}_{1,2} = \frac{\langle e_{1,2} \rangle}{\langle (1 + m(s)) R_{1,2} \rangle}.$$

The calibration factor (m) is determined by sample 4 of the GREAT3 HSC-like simulations (Mandelbaum et al. 2018).

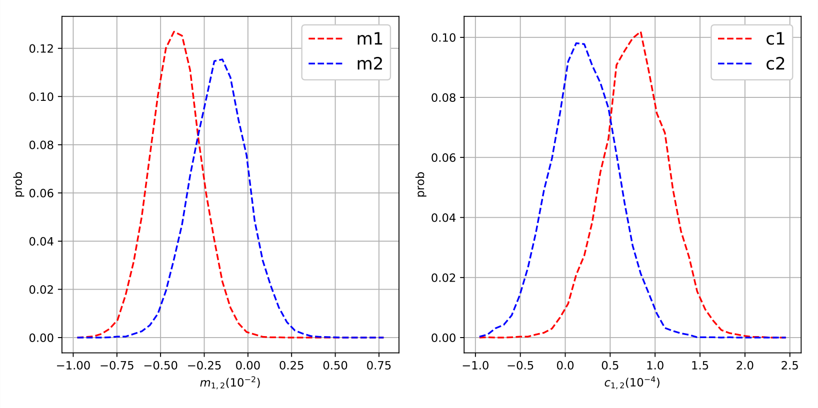


Calibration uncertainty

Sub-Sample with different PSF properties

sample	$\delta m(10^{-2})$	$\delta c(10^{-4})$
1st quartile	0.88 ± 0.56	-2.15 ± 1.48
2nd quartile	-0.36 ± 0.30	2.03 ± 0.73
3rd quartile	-0.49 ± 0.54	1.83 ± 1.35
4th quartile	-0.70 ± 0.86	-1.33 ± 2.23

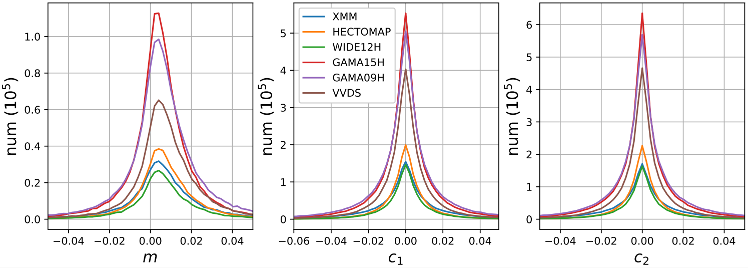
Bootstrap

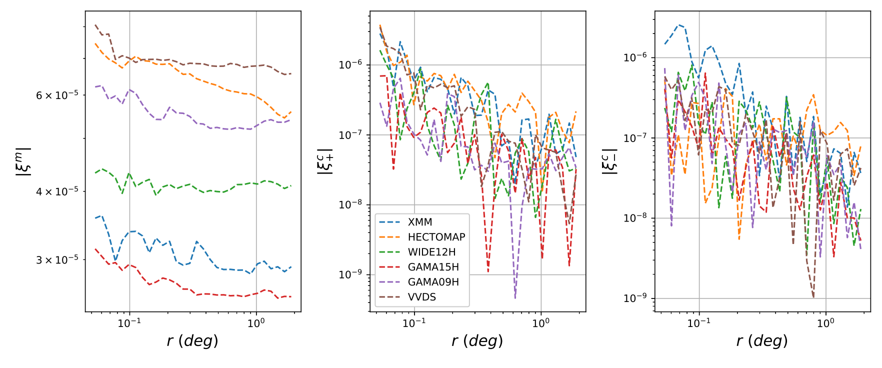

m1
m2
prob
0.12
0.10
0.08
0.06
0.04
0.02
0.00
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
$m_{1,2}(10^{-2})$
c1
c2
prob
0.10
0.08
0.06
0.04
0.02
0.00
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
$c_{1,2}(10^{-4})$

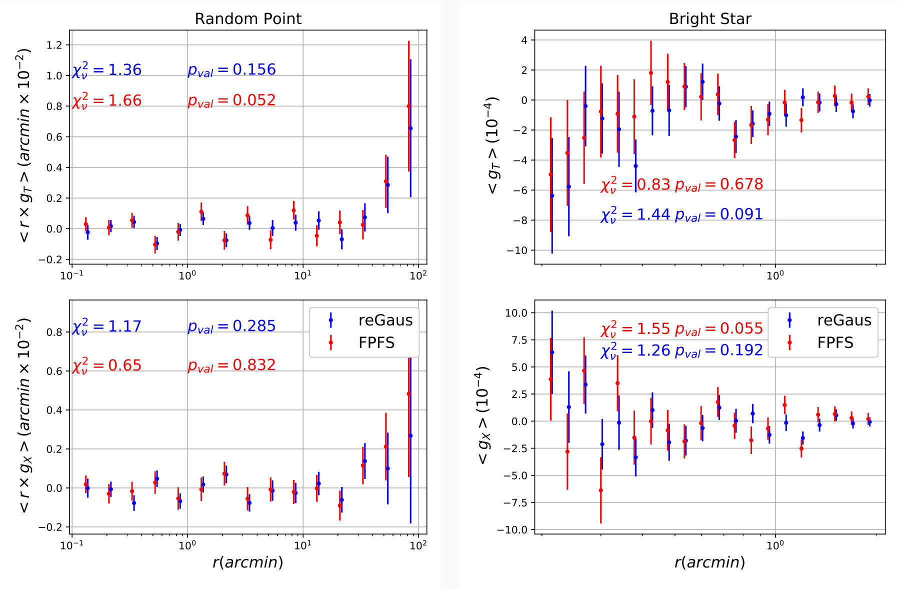
Application to Hyper Suprime-Cam (HSC)

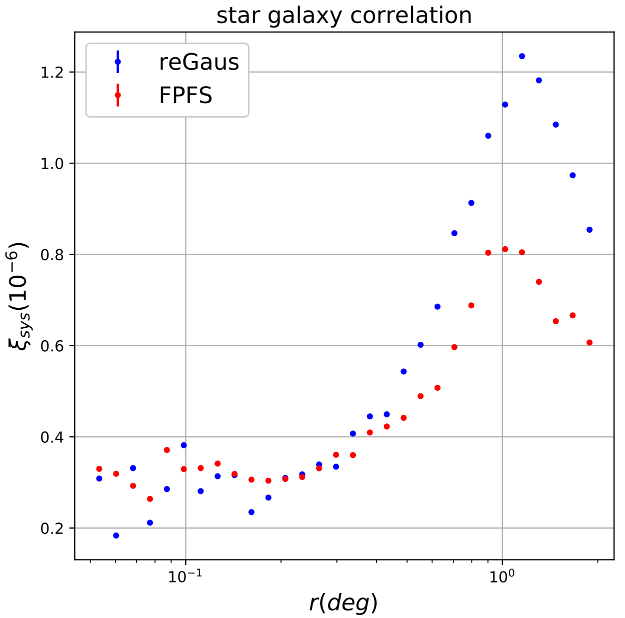
According to Lu et. al (2016),

- Choose bright stars, and the corresponding PSF;
- Simulate galaxies distorted by input shears;
- Convolve with star image and deconvolve with PSF image;
- estimate input shear and get biases m and $c_{1,2}$.



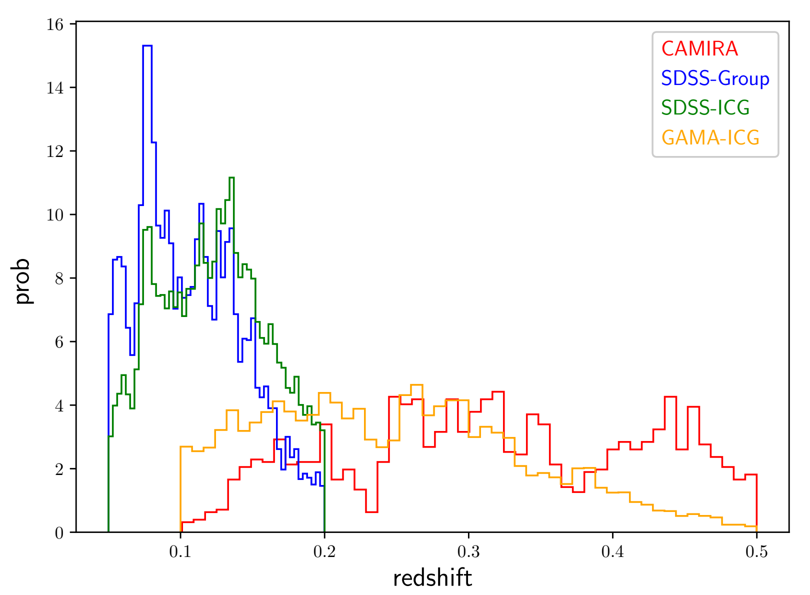

$|\xi^m|$
6×10^{-5}
4×10^{-5}
3×10^{-5}
10^{-1}
10^{0}
r (deg)
$|\xi^c_+|$
10^{-6}
10^{-7}
10^{-8}
10^{-9}
XMM
HECTOMAP
WIDE12H
GAMA15H
GAMA09H
VVDS
$|\xi^c_-|$


Random Point
Bright Star
$\chi^2_\nu = 1.36$ $p_{val} = 0.156$
$\chi^2_\nu = 1.66$ $p_{val} = 0.052$
$\chi^2_\nu = 0.83$ $p_{val} = 0.678$
$\chi^2_\nu = 1.44$ $p_{val} = 0.091$
$\chi^2_\nu = 1.17$ $p_{val} = 0.285$
$\chi^2_\nu = 0.65$ $p_{val} = 0.832$
$\chi^2_\nu = 1.55$ $p_{val} = 0.055$
$\chi^2_\nu = 1.26$ $p_{val} = 0.192$
reGaus
FPFS
$< r \times g_T > (arcmin \times 10^{-2})$
$< r \times g_X > (arcmin \times 10^{-2})$
$< g_T > (10^{-4})$
$< g_X > (10^{-4})$
$r(arcmin)$

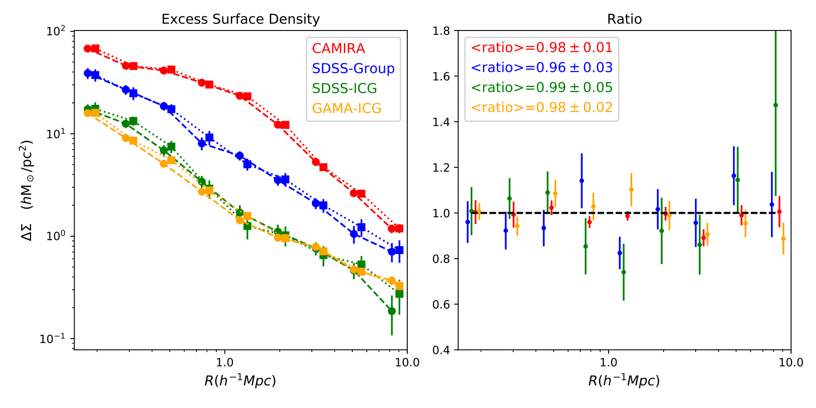


star galaxy correlation
reGaus
FPFS
$\xi_{sys}(10^{-6})$
$r(deg)$
1.2
1.0
0.8
0.6
0.4
0.2
10^{-1}
10^{0}

Lens catalogs

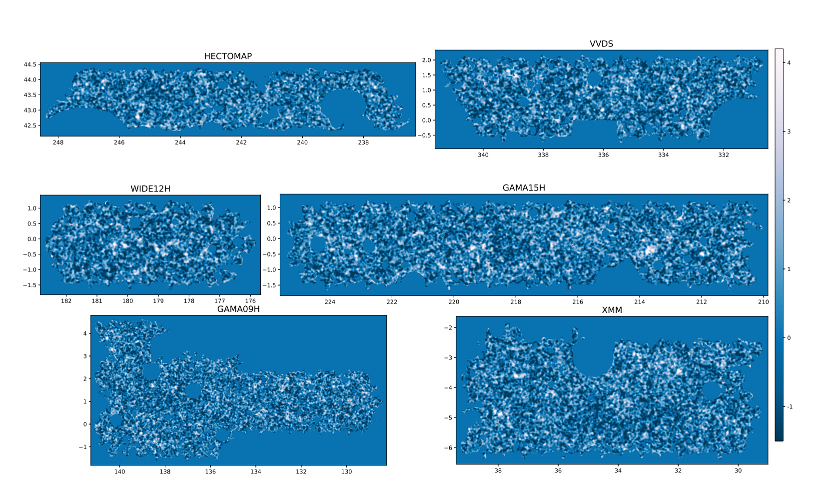


Results

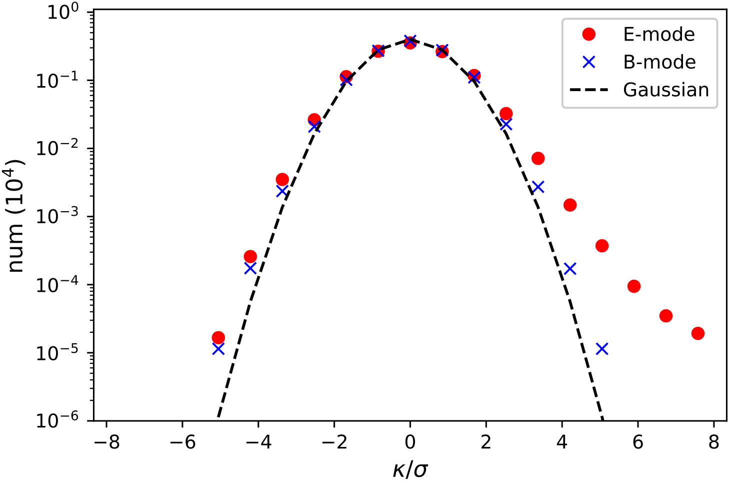

Excess Surface Density
CAMIRA
SDSS-Group
SDSS-ICG
GAMA-ICG
$\Delta\Sigma \quad (hM_\odot/pc^2)$
10^2
10^1
10^0
10^{-1}
1.0
10.0
$R(h^{-1}Mpc)$
Ratio
<ratio>=0.98 ± 0.01
<ratio>=0.96 ± 0.03
<ratio>=0.99 ± 0.05
<ratio>=0.98 ± 0.02
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
1.0
10.0
$R(h^{-1}Mpc)$

Mass map

mass map



PDF of E(B)-mode



Thanks