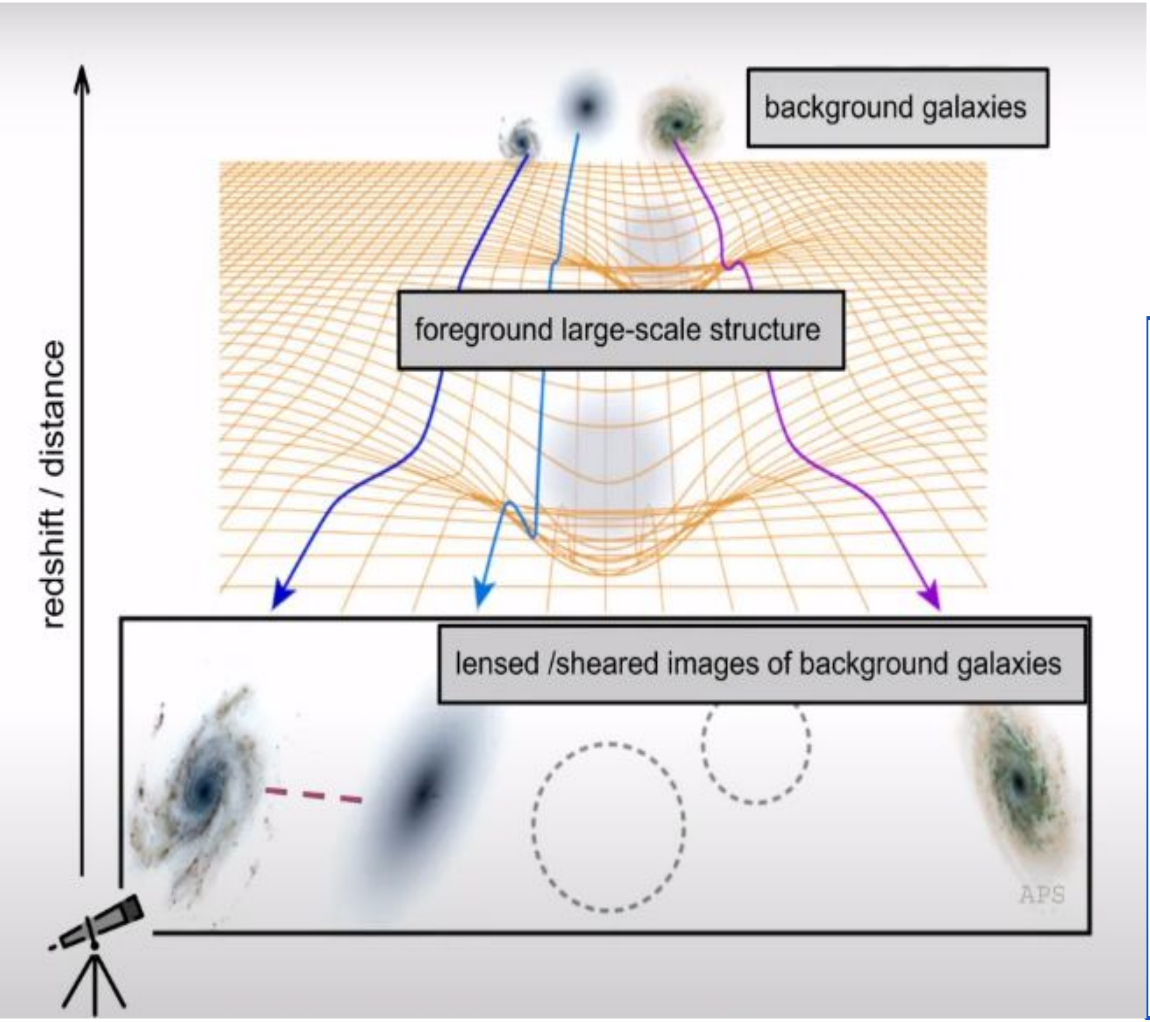
Cosmic Shear Analysis from Images to Cosmology

Xiangchong Li (CMU)



$$\mathbf{A} = \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}$$

where $g_{1,2} \sim 0.03, \ \sigma_g \sim 0.26$

1. Sensitive to the total mass along the line of sight;
2. Shear is spin-2 and small;

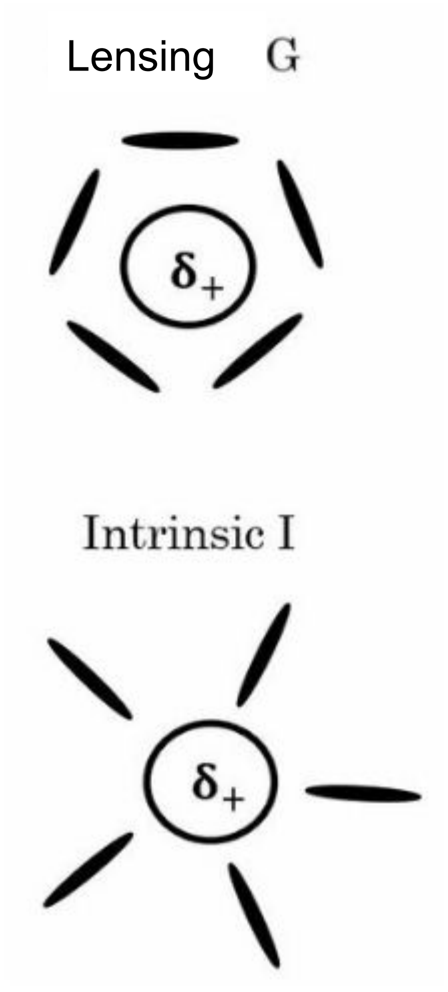
$$\hat{g}_i = R_{ij} g_j + \mathcal{O}(g^3)$$

3. Shear is a coordinate distortion; no need to know galaxy morphology.

[Li & Mandelbaum (2022)]()

Galaxies are intrinsic aligned but it has different patterns to the lensing shear distortion.

Intrinsic Alignment is take into the modeling of two-point statistics of galaxy shape.



$$\mathbf{A} = \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}$$

where $g_{1,2} \ll 1$

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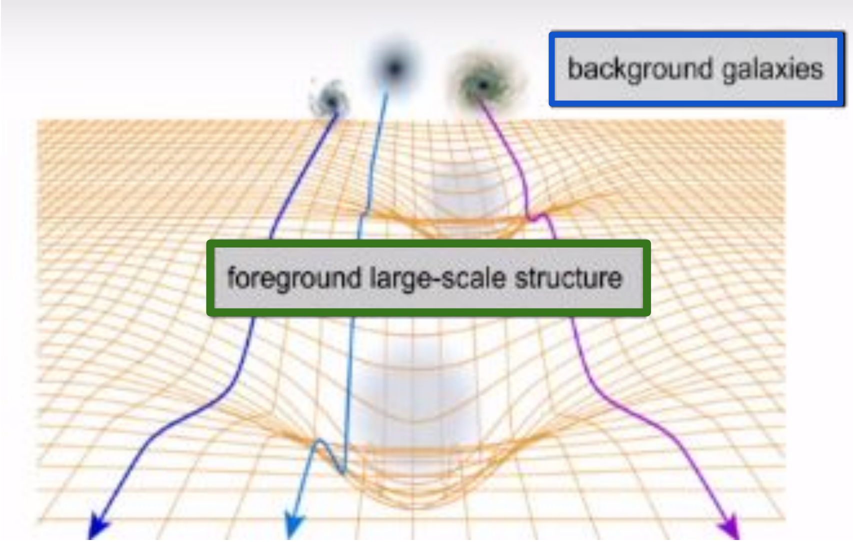
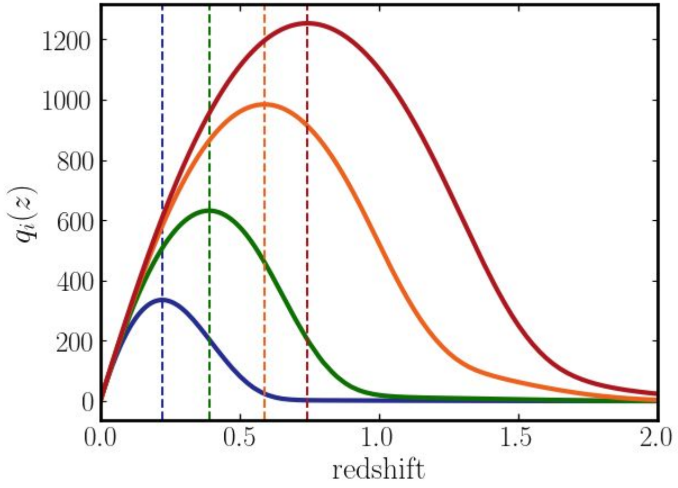
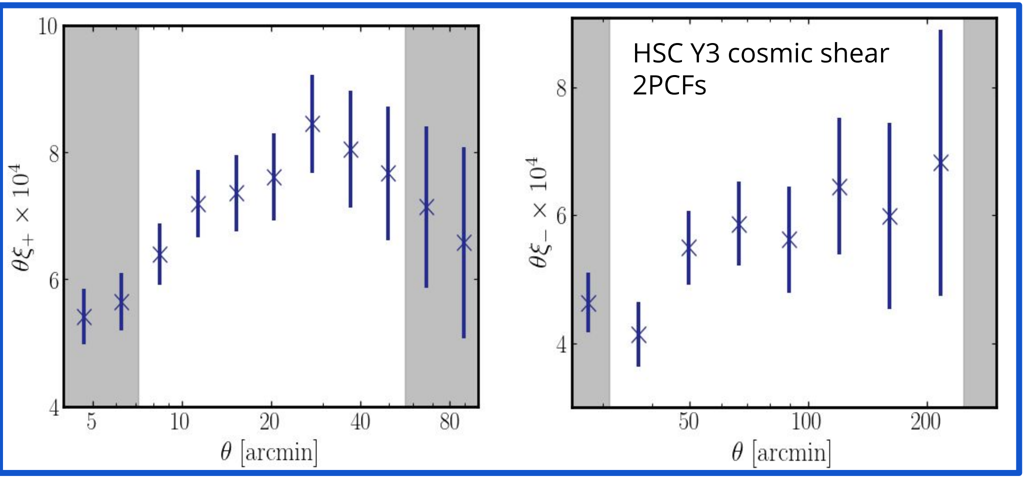
[Li & Mandelbaum (2022)]





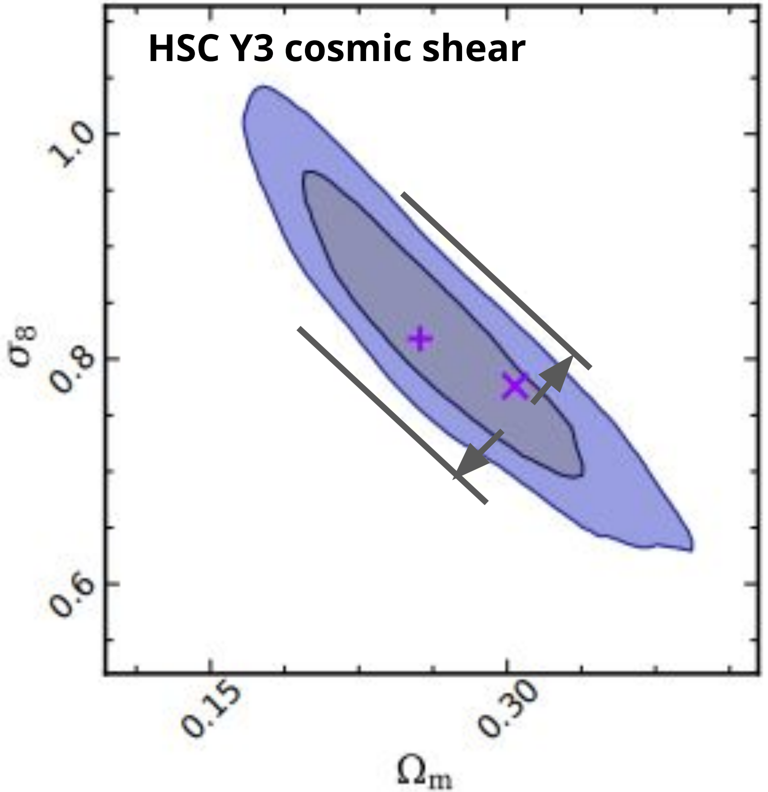
$$q_i(\chi) = \frac{3}{2}\Omega_{\mathrm{m}} \left(\frac{H_0}{c}\right)^2 \frac{\chi}{a(\chi)} \int_{\chi}^{\chi_H} d\chi' n_i(\chi') \frac{\chi' - \chi}{\chi'},$$



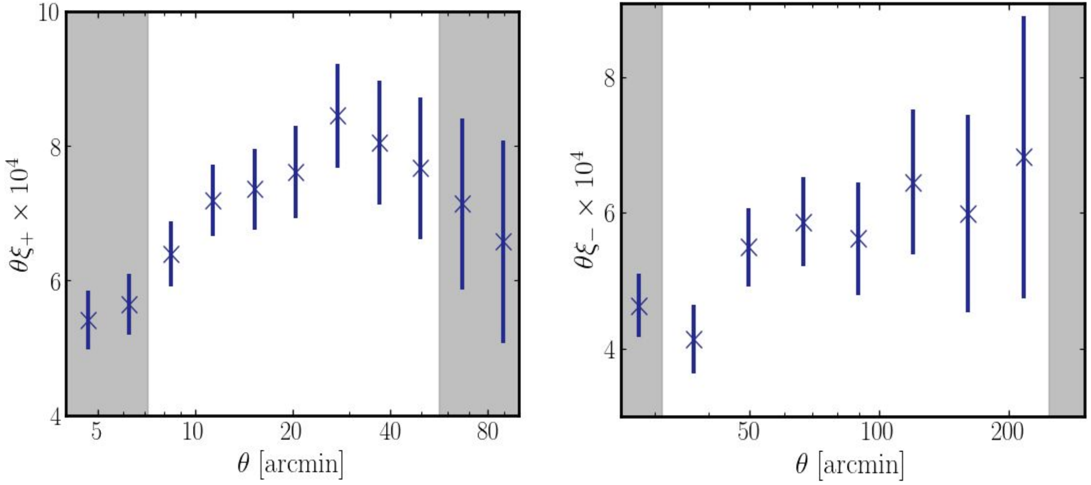
$$\boxed{\xi^{ij}_{+/-}(\theta)} = \frac{1}{2\pi}\int d\ell\, \ell J_{0/4}(\theta\ell) \left(C^{E;ij}(\ell) \pm C^{B;ij}(\ell)\right),$$

$$C^{ij}_{\mathrm{GG}}(\ell) = \int_0^{\chi_H} d\chi \frac{q_i(\chi) q_j(\chi)}{\chi^2} \boxed{P_{\mathrm{m}}\left(k = \frac{\ell + 1/2}{\chi}; \chi\right)},$$

σ_8 and Ω_{m}



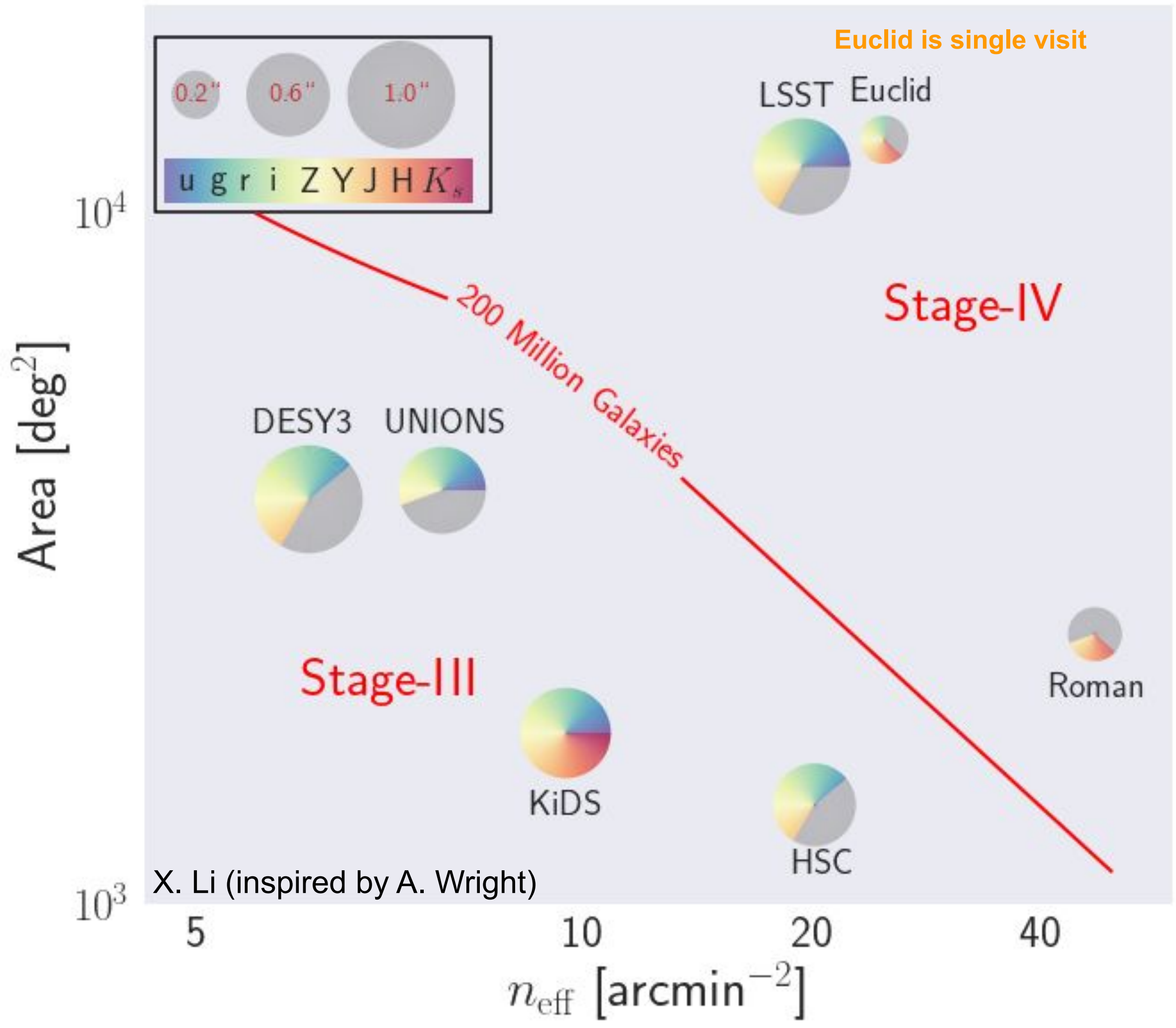
+: Marginalized 1D Max
x: Maximum in high-dimensional space (MAP)



Shape-shape correlation functions

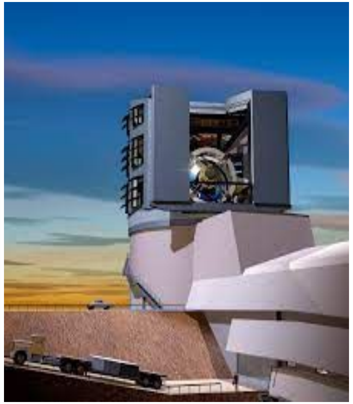
$$S_8 = \sigma_8(\Omega_{\rm m}/0.3)^{0.5}$$

Current Weak-lensing cosmic shear cannot distinguish **strong clustering with small matter density** and **weak clustering with large matter density**



Ground based

LSST



Space based

Euclid

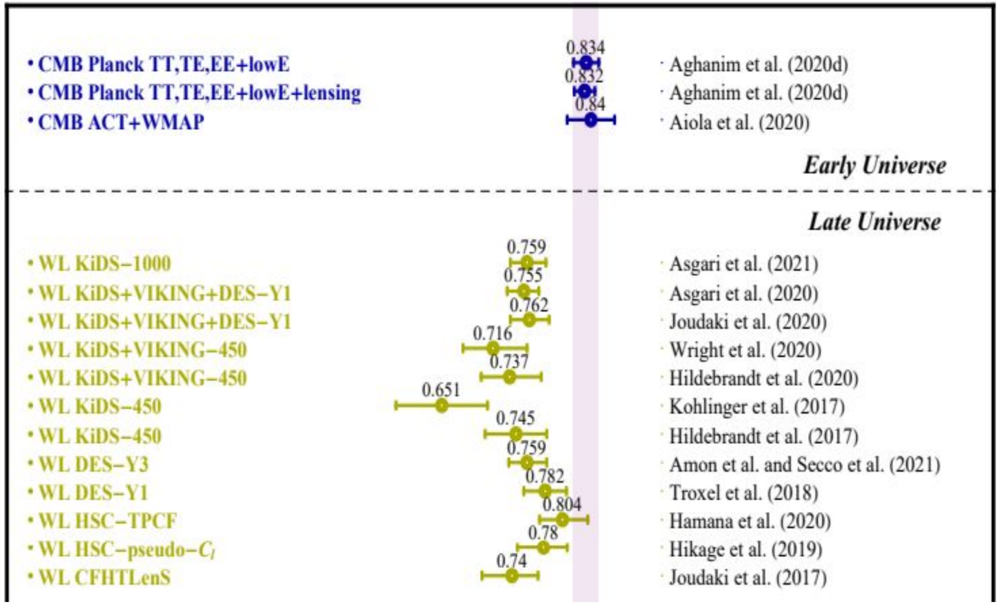
Balloon based (Future)

SuperBIT

For stage-IV, we will have much smaller contour. It is more challenging for systematics control.

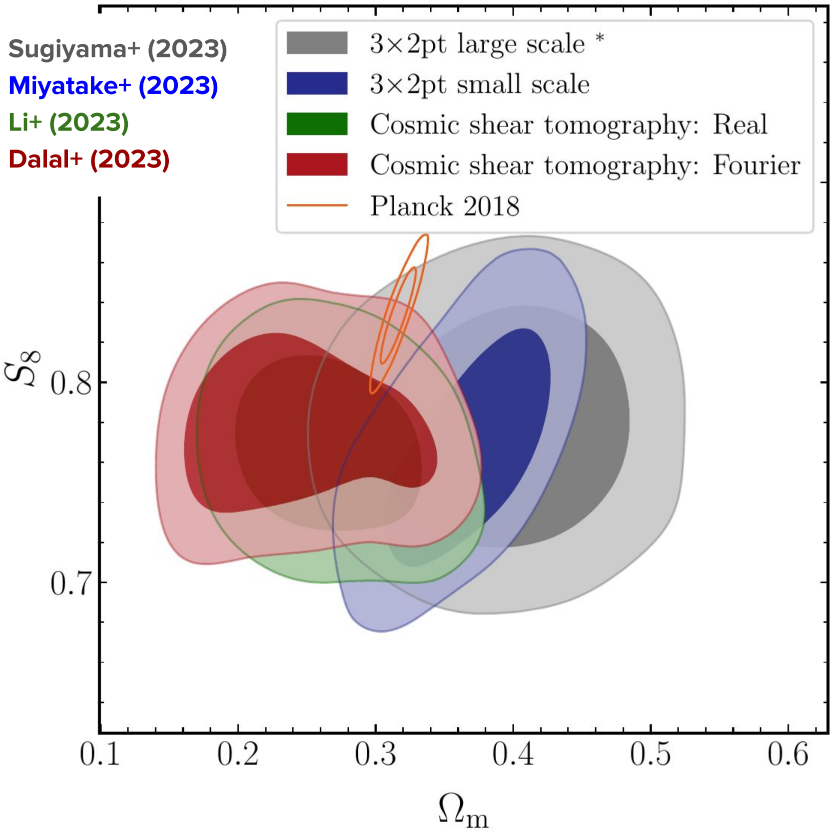
S_8 tension?

Weak lensing (Stage III) probes prefer smaller S_8 compared to **CMB**, if we assume ΛCDM is correct.



SNOWMASS 2021 Summer study — Abdalla et al. (2022)

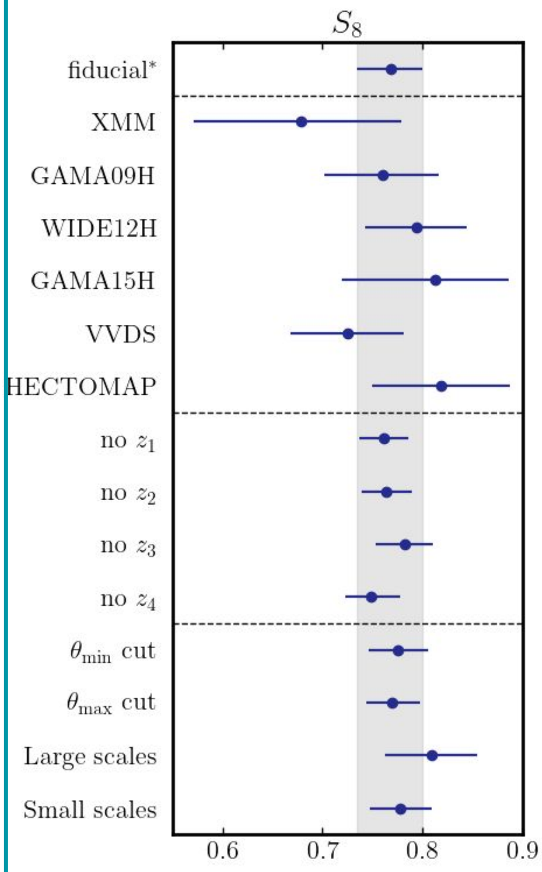
HSC Y3 Weak Lensing Cosmology



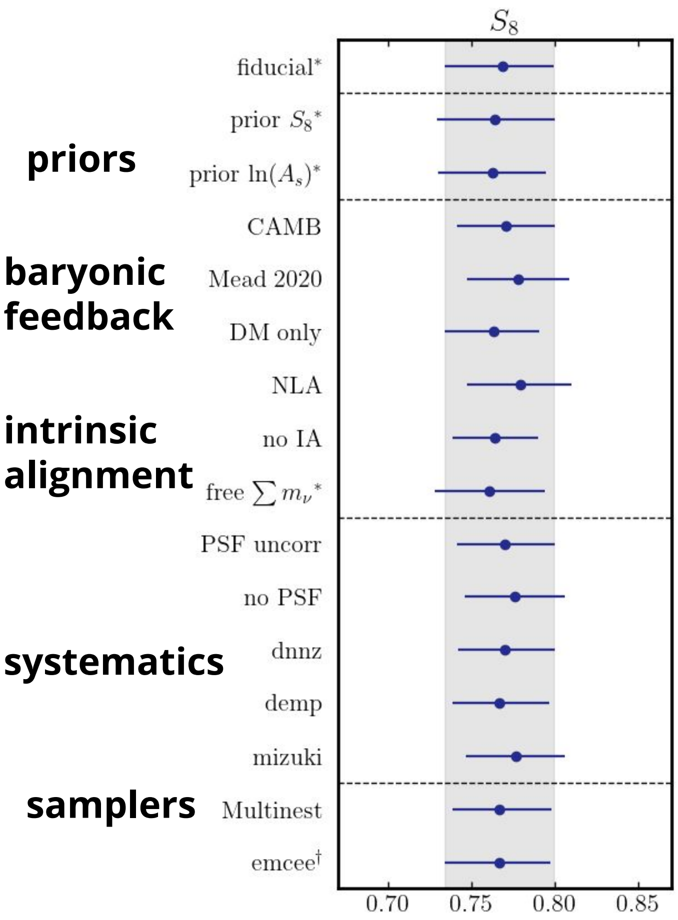
For each analysis, tests are done on three blinded data sets (one of them is the truth), and the analysors do not know which one is the truth.

In addition, all the plots are shifted to centered at zero before unblinding

Internal Consistency

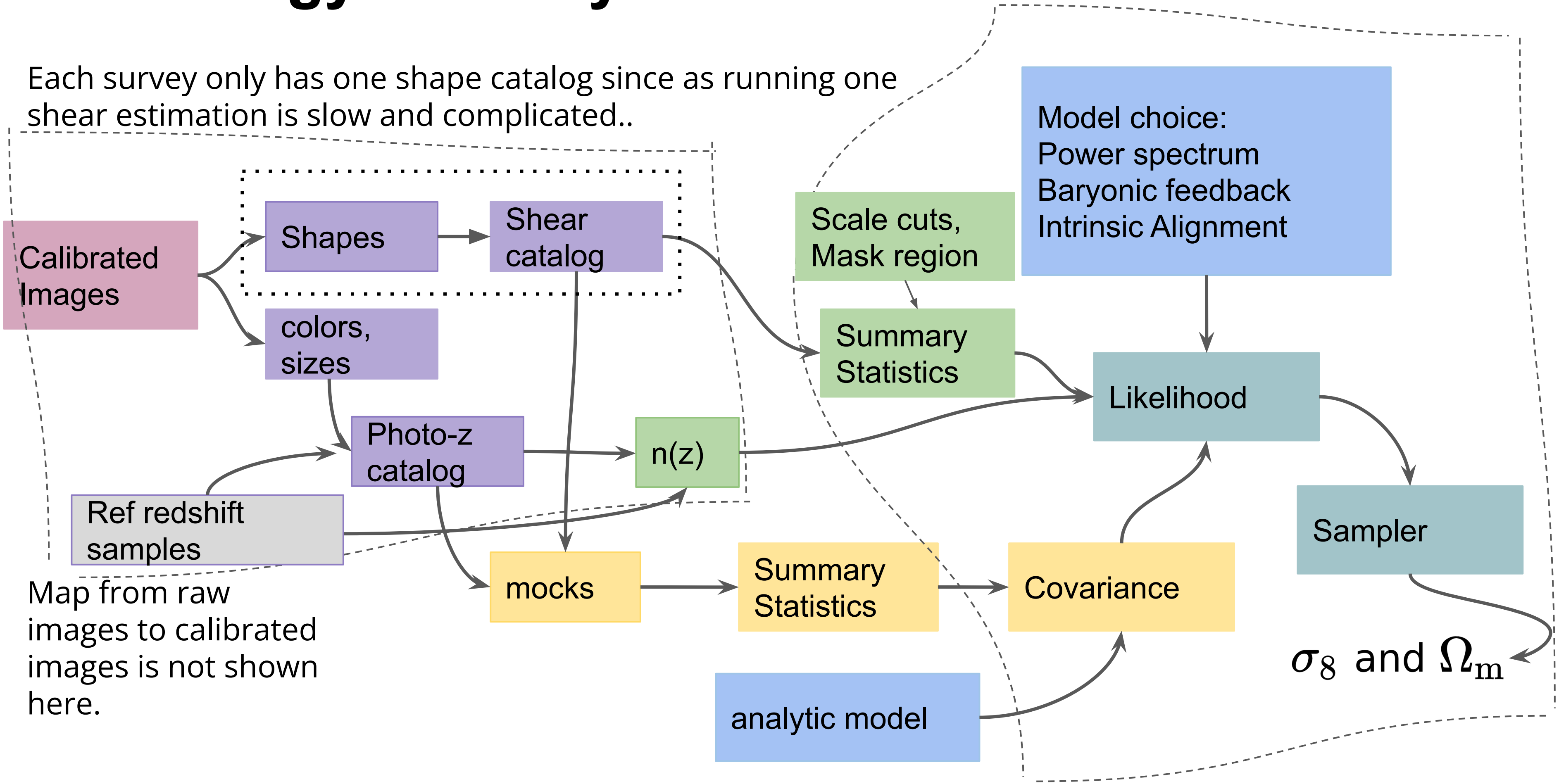


Different Models



Cosmology Industry

Each survey only has one shape catalog since as running one shear estimation is slow and complicated..

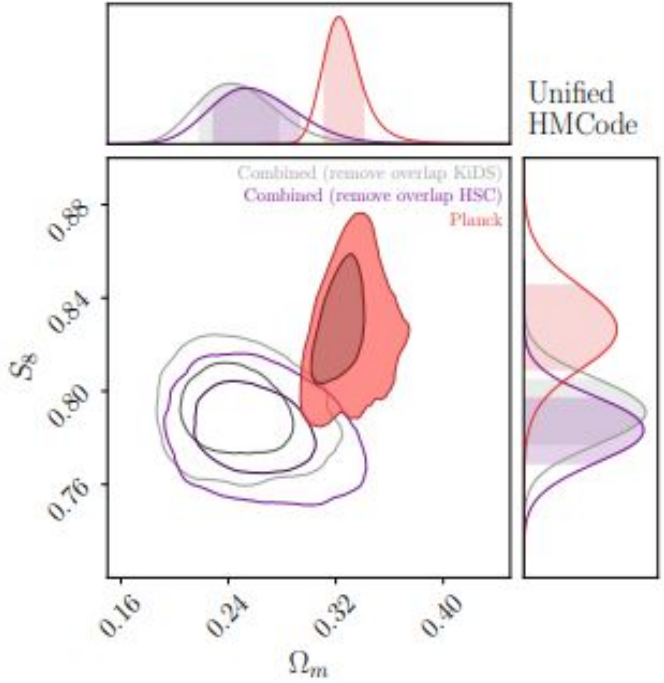


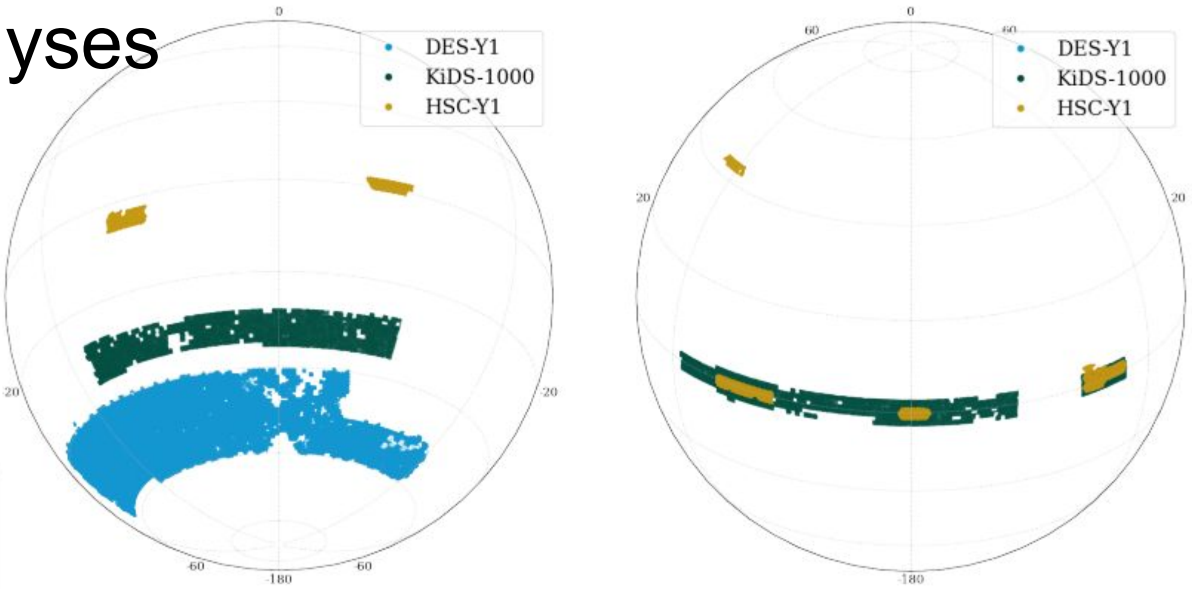
Map from raw images to calibrated images is not shown here.

Stage-III joint analyses

Longley et. al (2022)

Error on S8 improved from 0.026 to 0.016





Catalog level joint analysis
Combining information (joint shear estimation) from image level is very complicated and slow.
As a result, **overlapped regions have to be removed**.
This does not work for **stage-IV** surveys since the areas are largely covered between them.
(see also Abbott et. al (2023))

Measuring Shear Distortion

intrinsic gal $\boxed{\bar{f}(\bar{\mathbf{x}})} = \boxed{f(\mathbf{x})}$ **lensed gal**

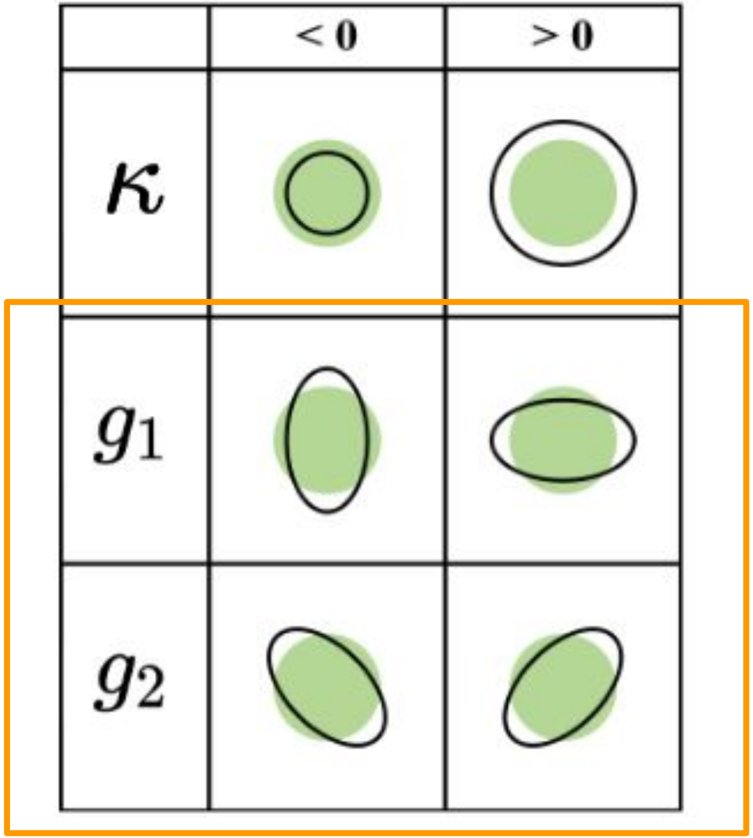
$$\bar{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

$$\mathbf{A} = \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}$$

$$\text{where } g_{1,2} \ll 1$$

Assumptions:
(i) intrinsic galaxies are randomly oriented;
(ii) image noise is symmetric to zero.

Why current shear estimation takes so much resource?



Metacalibration to estimate linear shear response

We begin from linear observable and will generalize to non-linear observables.

Observable ν: projection of lensed galaxy onto a basis —

$$\nu = \int f(\mathbf{x})\chi(\mathbf{x})\mathrm{d}^2x$$

$$= \int \bar{f}(\mathbf{Ax})\chi(\mathbf{x})\mathrm{d}^2x$$

How does ν transform under a shear distortion?

Huff & Mandelbaum (2017), Sheldon & Huff (2017):

$$\nu = \bar{\nu} + g_1 \boxed{\frac{\partial \nu}{\partial g_1}} + g_2 \boxed{\frac{\partial \nu}{\partial g_2}}$$

$$\boxed{\frac{\partial \nu}{\partial g_i}} = \frac{\partial\left(\int \bar{f}(\mathbf{Ax})\chi(\mathbf{x})\mathrm{d}x^2\right)}{\partial g_i}$$

Shear response can be estimated by adding small shear distortions to galaxy image.

A passive line of thought: distortions on bases

$\vec{x} \to \mathbf{J}\vec{x}$

$\mathbf{J} = \mathbf{A}^{-1}$

$$\nu = \int \boxed{\bar{f}(\mathbf{A}\mathbf{x})}\chi(\mathbf{x})\mathrm{d}^2 x$$

$$\nu \propto \int \bar{f}(\mathbf{x})\boxed{\chi(\mathbf{J}\mathbf{x})}\mathrm{d}^2 x$$

If we ***fix the galaxy images but distort the bases***, the measurement is **faster**, since we can pre-derive (compute) the **shear responses of the bases**.

$$\frac{\partial \nu}{\partial g_i} \propto \int \bar{f}(\mathbf{x}) \boxed{\frac{\partial \chi(\mathbf{J}\mathbf{x})}{\partial g_i}} \mathrm{d}^2 x$$

Chain rule for derivatives

$$\boxed{\begin{aligned} \frac{\partial \chi(\mathbf{J}\mathbf{x})}{\partial g_1} &= \left(x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y} \right) \chi(\mathbf{x}) \\ \frac{\partial \chi(\mathbf{J}\mathbf{x})}{\partial g_2} &= \left(x\frac{\partial}{\partial y} + y\frac{\partial}{\partial x} \right) \chi(\mathbf{x}) \end{aligned}}$$

Raising and Lowering Operators (Refregier 2001)

$$\hat{a_1} = x + \frac{\partial}{\partial x} \qquad \hat{a_2} = y + \frac{\partial}{\partial y}$$

$$\hat{a_1}^\dagger = x - \frac{\partial}{\partial x} \qquad \hat{a_2}^\dagger = y - \frac{\partial}{\partial y}$$

$$\boxed{\begin{aligned} \frac{\partial \chi(\mathbf{J}\mathbf{x})}{\partial g_1} &= \frac{1}{2}\left(\hat{a_1}^2 - (\hat{a_1}^\dagger)^2 - \hat{a_2}^2 + (\hat{a_2}^\dagger)^2\right)\chi(\mathbf{x}) \\ \frac{\partial \chi(\mathbf{J}\mathbf{x})}{\partial g_2} &= \frac{1}{2}\left(\hat{a_1}\hat{a_2} - \hat{a_1}^\dagger \hat{a_2}^\dagger\right)\chi(\mathbf{x}) \end{aligned}}$$

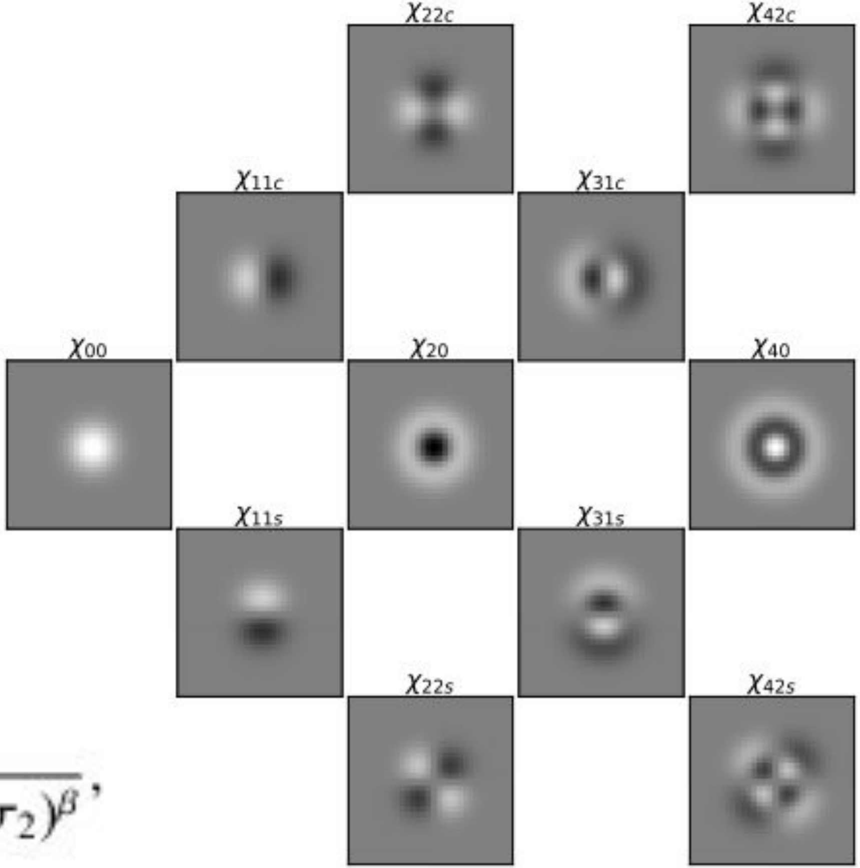
If we use eigen functions of Quantum harmonic oscillator as our basis, shear distortion only couples a finite number of basis.

Shapelet Bases: Eigen functions of 2D QHO

Massey & Refregier (2005):

$$M_{nm} = \int f(\mathbf{x})\chi_{nm}\,\mathrm{d}^2x$$

- 'n' : **radial profile**;
- 'm' : **spin**;
- 'c' refers to **real part** (cosine);
- 's' refers to **imaginary part** (sine).



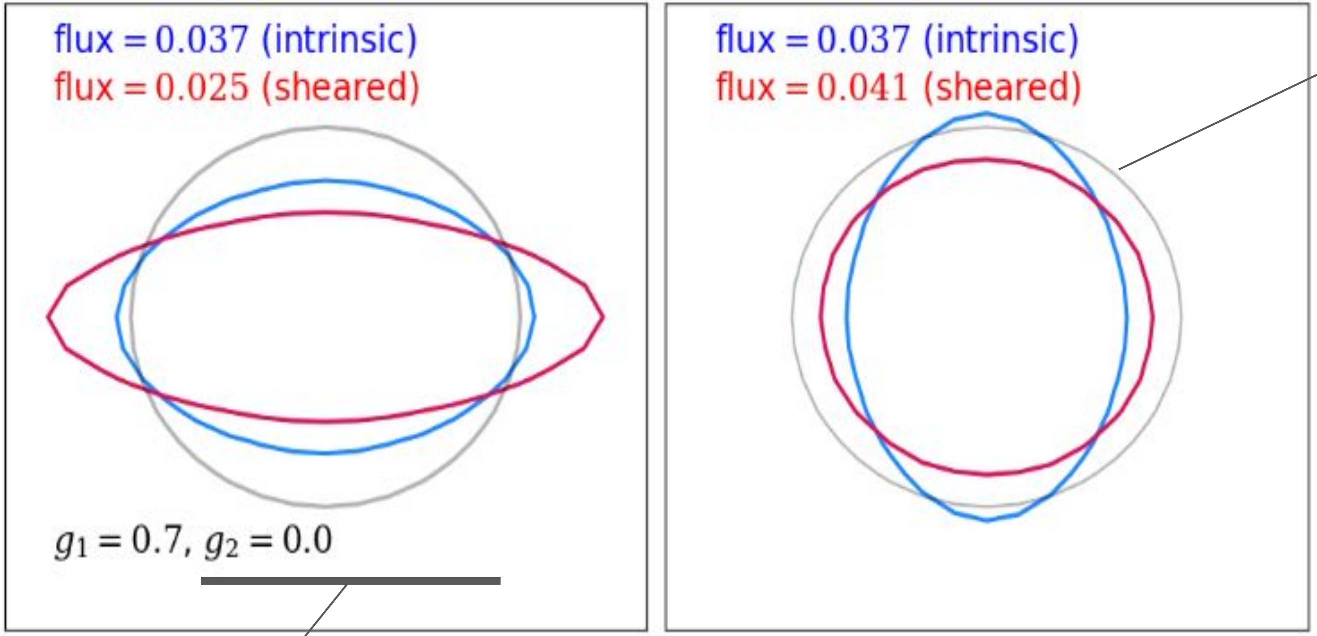
Nonlinear Observable: Ellipticity

$$e_1 + \mathrm{i}\,e_2 \equiv \frac{M_{22}}{(M_{00} + c_0\sigma_0)^{\alpha}(M_{00} + M_{20} + c_2\sigma_2)^{\beta}},$$

Anisotropy Caused by Selections

Kaiser (2000)

The following plot gives an example of selection bias caused by a cut on aperture flux



Aperture

As a result, we are more likely to select galaxies that are **anti-aligned** with the shear

We need analytic correction for this !

The shear stretches image in the horizontal direction. The left galaxy (aligned with the shear) appears to be fainter; while the right (anti-aligned with the shear) appears to be brighter.

Selection is a weighting with smooth step function

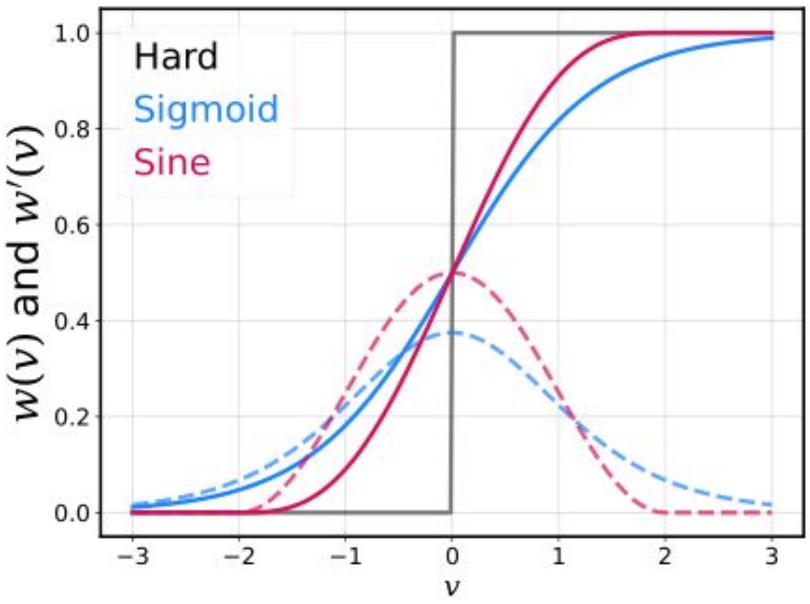
Shear estimation with average ellipticity $\langle w e_\alpha \rangle$

(spin-0) selection weight and (spin-2) ellipticity

$w(\nu_i)$ and $e_\alpha(\nu_i)$

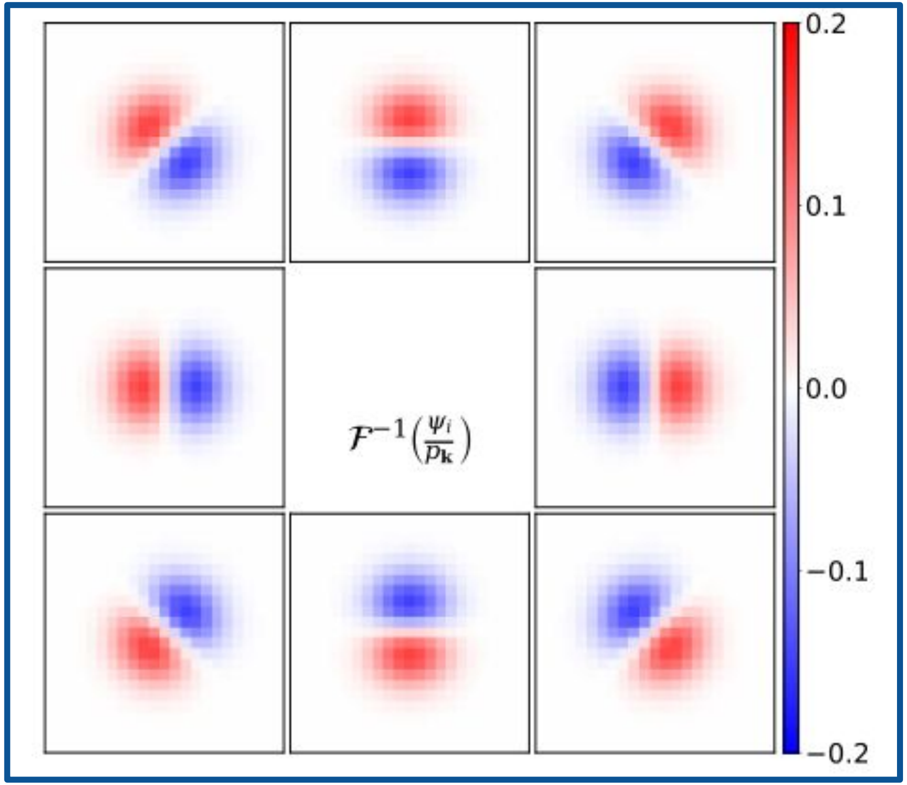
Both the weight and the ellipticity need to be differentiable to get the shear response.

$$w_2(v \,|\, \mu, \omega) = \begin{cases} 0 & \text{if } v \in (-\infty, \mu - \omega) \\ \frac{1}{2} + \frac{(v-\mu)}{2\omega} + \frac{1}{2\pi} \sin\left(\frac{(v-\mu)\pi}{\omega}\right) & \text{if } v \in [\mu - \omega, \mu + \omega] \\ 1 & \text{if } v \in (\mu + \omega, +\infty) \end{cases}$$

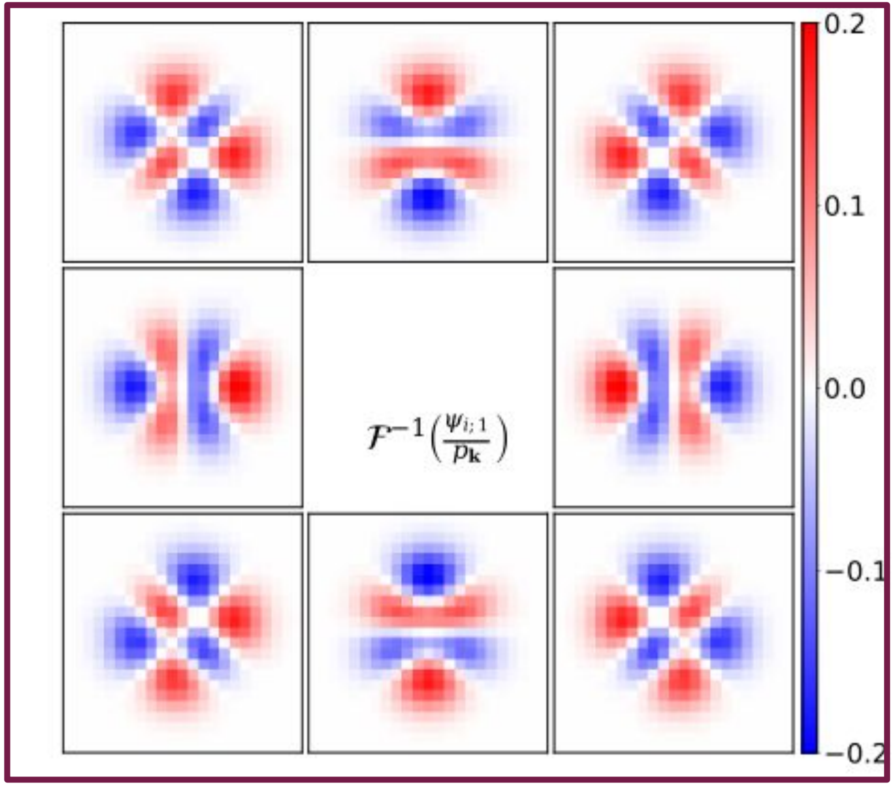


Peak Bases and their Shear Responses

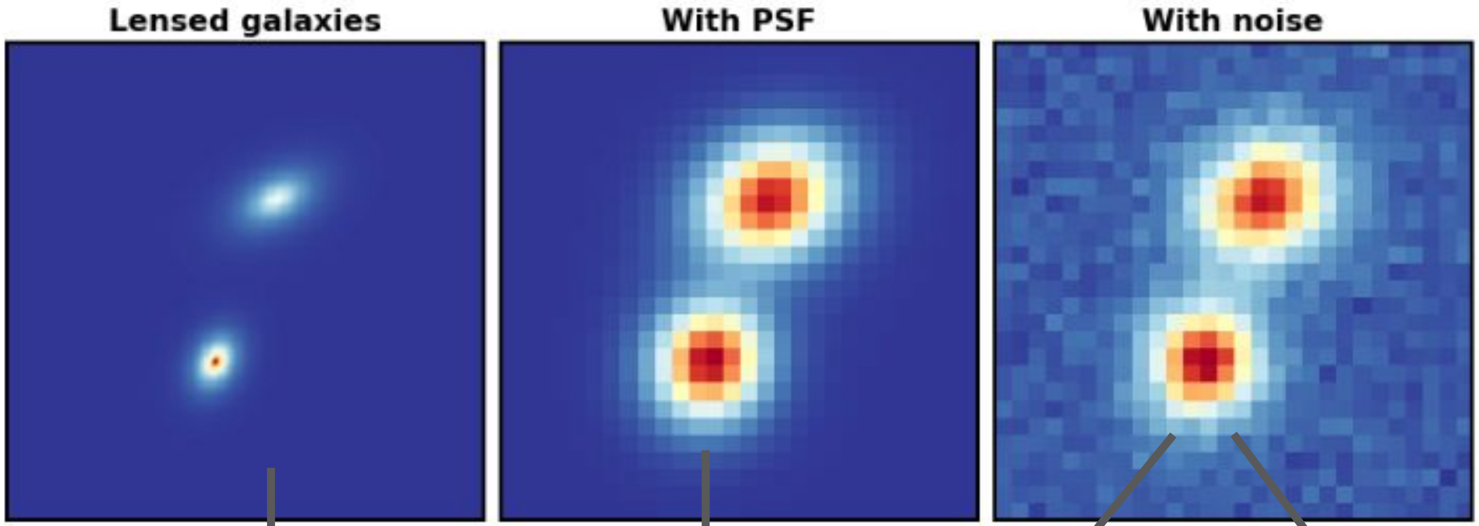
Bases that are used to detect peaks as galaxy candidates



Shear response of those bases.



Systematics in shear measurement



$$g \sim 0.03$$

$$g^2 \sim 10^{-3}$$

Lensed gals: ν , e

PSF smearing: $p_{\mathbf{x}}$

noise bias $\langle \tilde{e} \rangle \neq \langle e \rangle$

measurement error: $\boldsymbol{\delta\nu}$, $\boldsymbol{\delta e}$

$$\hat{g}_{1,2} = (1 + \boxed{m})g_{1,2} + \boxed{c_{1,2}} + \mathcal{O}(g^3)$$

multiplicative bias **additive bias**

LSST req.

$$m < 3 \times 10^{-3}$$

PSF revision: Deconvolution in Fourier space

$$f^p(\mathbf{x}) = \int f(\mathbf{x}) p(\mathbf{x} - \mathbf{x}') \mathrm{d}^2 x'$$

$$f_{\mathbf{k}} = \frac{f^p_{\mathbf{k}}}{\boxed{p_{\mathbf{k}}}}$$

Fourier transform of PSF

Then project on bases (i.e., shapelet and peak bases):

$$M_{nm} = \int \chi^*_{nm} \frac{f^p_{\mathbf{k}}}{p_{\mathbf{k}}} \mathrm{d}^2 k$$

Note: scale radius of the basis functions should be larger than the scale radius of PSF in configuration space (equivalently, smaller in Fourier space) to avoid noise on small scales being boosted by PSF deconvolution.

Noise is a Perturbation on Galaxy Number Distribution

Noise bias correction:

$$\langle \boxed{\tilde{w}\tilde{e}_\alpha} \rangle = \langle w e_\alpha \rangle + \frac{1}{2}\sum_{i,j}\left\langle \boxed{\frac{\partial^2 \left(\tilde{w}\tilde{e}_\alpha\right)}{\partial v_i \partial v_j}} \boxed{\delta v_i \delta v_j} \right\rangle + \mathcal{O}\left(\left(\delta v_i\right)^4\right)$$

Noisy observables

Independent of gals

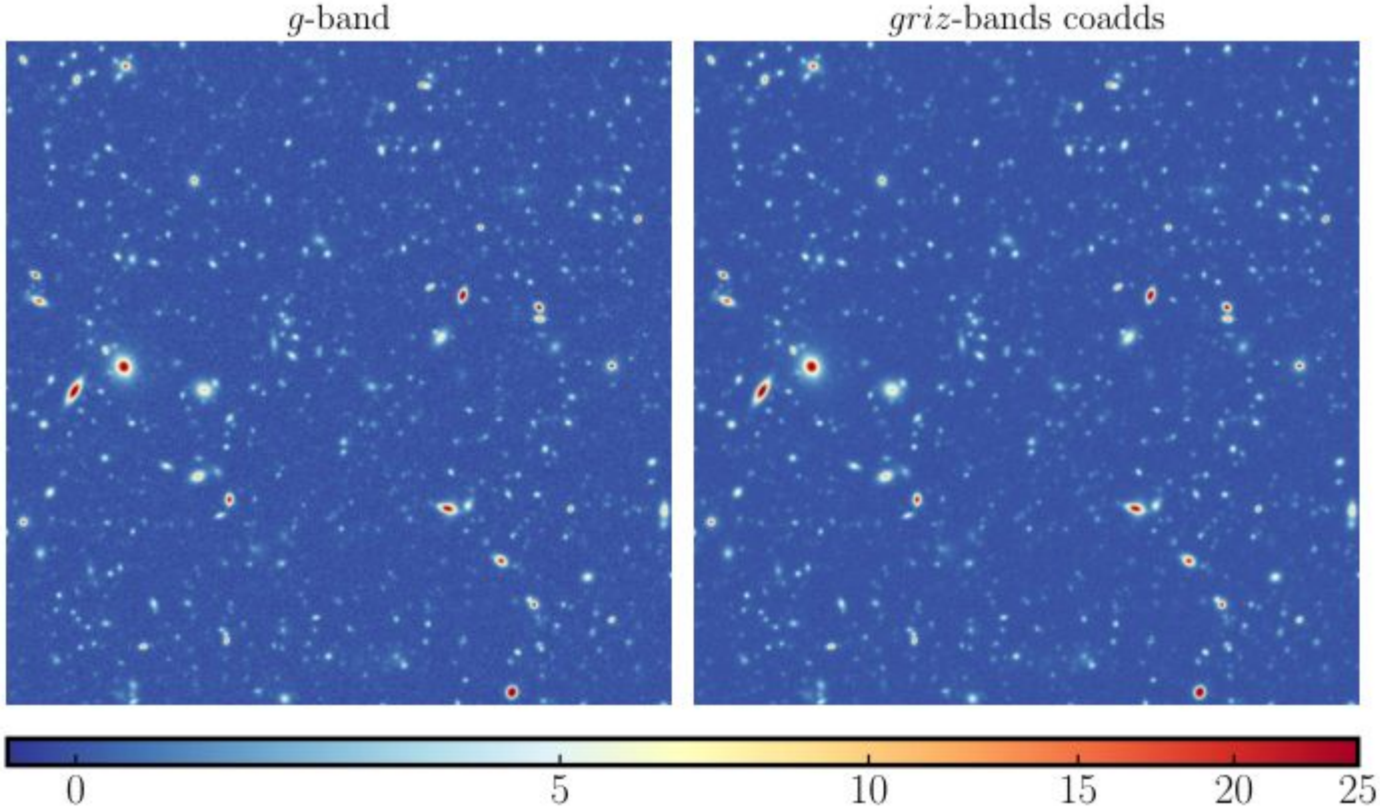
The leading order perturbation is a contraction between Hessian matrix (noise response) and covariance matrix of measurement error on the linear observables.

Under what condition the analytic covariance of measurement errors from image noise can be derived:

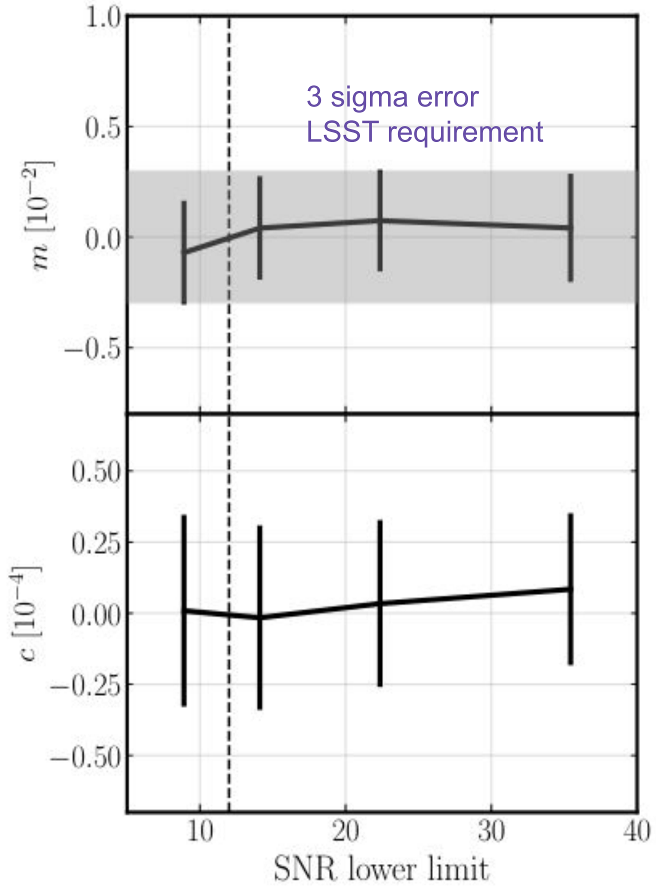
	not correlated	correlated
Homogeneous	✅	✅
inhomogeneous	✅	❌

(correlated and homogeneous are defined in configuration space.)

Test on LSST image simulations (Sheldon et. al (2023))



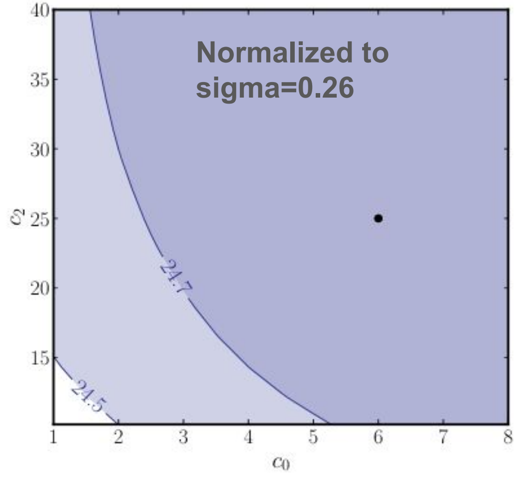
>1000 galaxies per cpu second

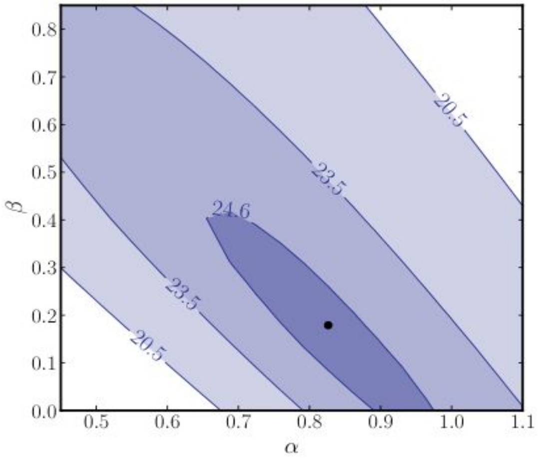


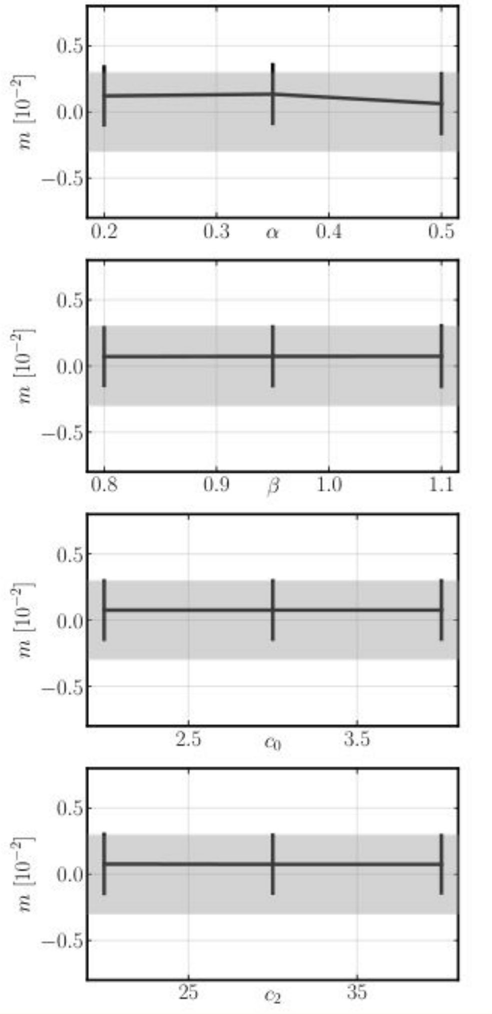
~4k cpu hours (allocation units) to measure shear from all the LSST Y10 images

Effective number density is ~25

Effective galaxy number density







Performance (accuracy) is stable for different hyperparameter setups.

For details see Li et. al (2018); Li, Li & Massey (2022) ; Li & Mandelbaum (2022)

Three Takeaways:

Analytic shear estimation is simple, accurate, and fast!l

1. Enable us to **directly** understand how different detection, deblending, shape estimation, n(z) influence cosmology constraint;
2. Optimization of constraints from image level;
3. Enable fast joint analysis between Stage-IV surveys.