Analytical Shear Responses of Galaxy properties and Detection

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Based on Li et. al (2018); Li, Li & Massey (2022) ;
Li & Mandelbaum (2022)

1. **Shear Responses of Galaxy properties**
2. Shear Response of Detection
3. PSF and Noise Corrections
4. Tests on Image Simulations

Shear Distortion

$$f_I(\vec{x}_I) = f(\vec{x})$$
$$\vec{x}_I = \mathbf{A}\vec{x}$$

$$\mathbf{A} = \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}$$

where $g_{1,2} \ll 1$

How to measure the shear distortion from galaxy images?

(assuming intrinsic galaxies are randomly oriented)

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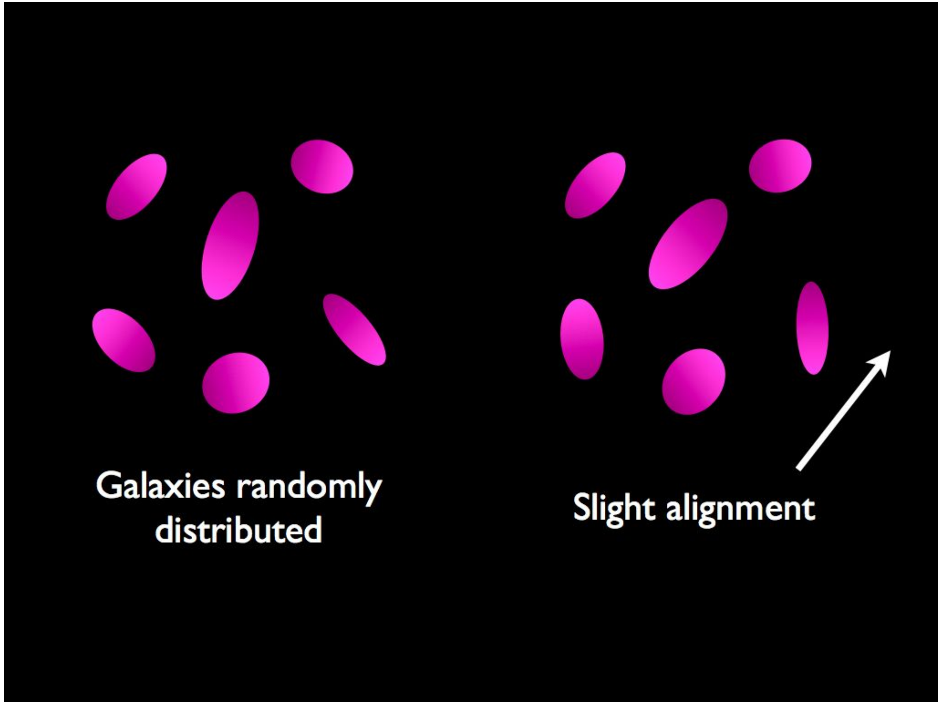

Image: E Grocutt, IfA, Edinburgh

How to measure the shear distortion from galaxy images?
(assuming intrinsic galaxies are randomly oriented)

Measuring Shear Distortion

intrinsic gal $\boxed{\bar{f}(\bar{\mathbf{x}})} = \boxed{f(\mathbf{x})}$ lensed gal

$$\bar{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

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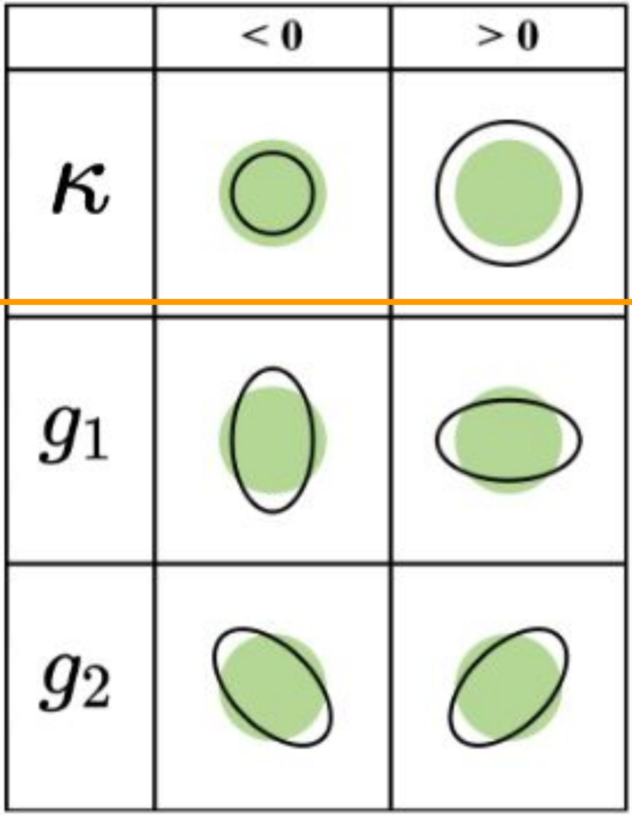
where $g_{1,2} \ll 1$

Develop an accurate analytic shear estimator even for blended gals after detection/selection.

Assumptions:

(i) intrinsic galaxies are randomly oriented;

(ii) image noise is symmetric to zero.



Metacalibration to estimate linear shear response

We begin from linear observable and will generalize to non-linear observables.

Observable ν: projection of lensed galaxy onto a basis —

$$\nu = \int f(\mathbf{x})\chi(\mathbf{x})\mathrm{d}^2 x$$

$$= \int \bar{f}(\mathbf{A}\mathbf{x})\chi(\mathbf{x})\mathrm{d}^2 x$$

How does ν transform under a shear distortion?

Huff & Mandelbaum (2017), Sheldon & Huff (2017):

$$\nu = \bar{\nu} + g_1 \boxed{\frac{\partial \nu}{\partial g_1}} + g_2 \boxed{\frac{\partial \nu}{\partial g_2}}$$

$$\boxed{\frac{\partial \nu}{\partial g_i}} = \frac{\partial\left(\int \bar{f}(\mathbf{A}\mathbf{x})\chi(\mathbf{x})\mathrm{d}x^2\right)}{\partial g_i}$$

Shear response can be estimated by adding small shear distortions to galaxy image.

A passive line of thought: distortions on bases

$\vec{x} \rightarrow \mathbf{J}\vec{x}$

$\mathbf{J} = \mathbf{A}^{-1}$

$$O = \int \boxed{\bar{f}(\mathbf{Ax})} \chi(\mathbf{x}) \mathrm{d}^2 x$$

$$O \propto \int \bar{f}(\mathbf{x}) \boxed{\chi(\mathbf{Jx})} \mathrm{d}^2 x$$

If we ***fix the*** ***galaxy images*** ***but distort the*** ***bases***, the measurement is **faster**, since we can pre-derive (compute) the **shear responses of the** **bases**.

$$\frac{\partial O}{\partial g_i} \propto \int \bar{f}(\mathbf{x}) \boxed{\frac{\partial \chi(\mathbf{Jx})}{\partial g_i}} \mathrm{d}^2 x$$

Chain rule for derivatives

$$\boxed{\begin{aligned} \frac{\partial \chi(\mathbf{Jx})}{\partial g_1} &= \left(x \frac{\partial}{\partial x} - y \frac{\partial}{\partial y} \right) \chi(\mathbf{x}) \\ \frac{\partial \chi(\mathbf{Jx})}{\partial g_2} &= \left(x \frac{\partial}{\partial y} + y \frac{\partial}{\partial x} \right) \chi(\mathbf{x}) \end{aligned}}$$

Raising and Lowering Operators (Refregier 2001)

$$\hat{a_1} = x + \frac{\partial}{\partial x} \qquad \hat{a_2} = y + \frac{\partial}{\partial y}$$

$$\hat{a_1}^\dagger = x - \frac{\partial}{\partial x} \qquad \hat{a_2}^\dagger = y - \frac{\partial}{\partial y}$$

$$\boxed{\begin{aligned} \frac{\partial \chi(\mathbf{Jx})}{\partial g_1} &= \frac{1}{2}\left(\hat{a_1}^2 - (\hat{a_1}^\dagger)^2 - \hat{a_2}^2 + (\hat{a_2}^\dagger)^2\right)\chi(\mathbf{x}) \\ \frac{\partial \chi(\mathbf{Jx})}{\partial g_2} &= \frac{1}{2}\left(\hat{a_1}\hat{a_2} - \hat{a_1}^\dagger \hat{a_2}^\dagger\right)\chi(\mathbf{x}) \end{aligned}}$$

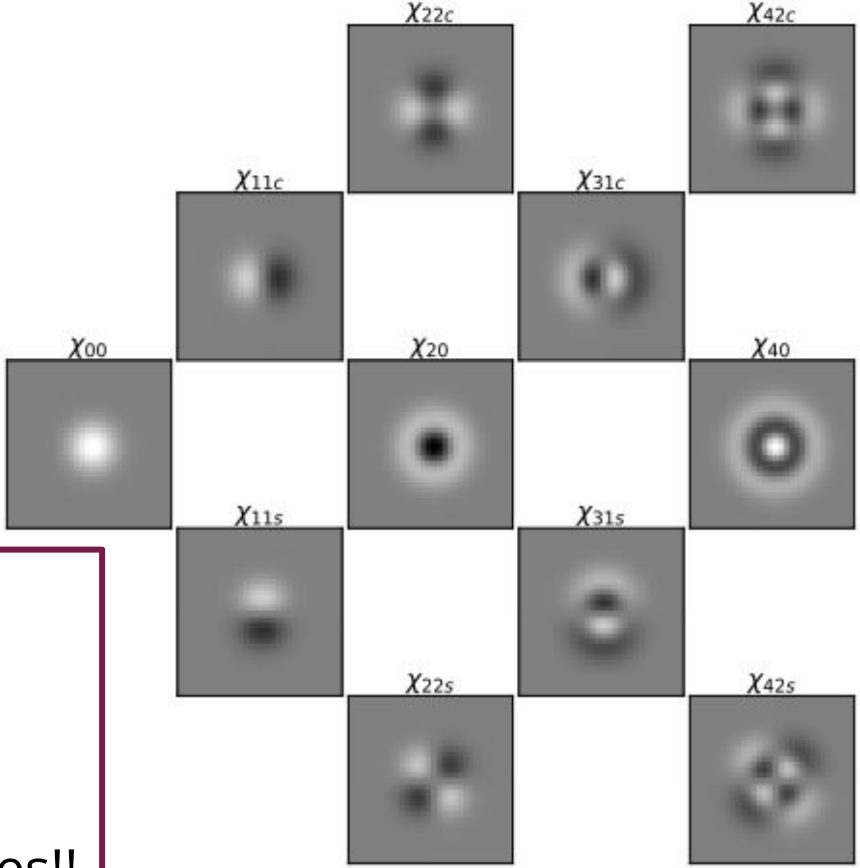
If we use the eigen function of Quantum harmonic oscillator as our basis, shear distortion only couples a finite number of basis.

Shaplet Bases: Eigen functions of 2D QHO

Massey & Refregier (2005):

$$M_{nm} = \int f(\mathbf{x}) \chi_{nm} \mathrm{d}^2 x$$

- ‘n' determines **radial profile**;
- ‘m' determines **spin**;
- ‘c' refers to **real part** (cosine);
- ‘s' refers to **imaginary part** (sine).



$$\frac{\partial \chi_{n,m}(\mathbf{Jx})}{\partial g_1} = a_1 \chi_{n-2,m-2} + a_2 \chi_{n+2,m-2} + a_3 \chi_{n-2,m+2} + a_4 \chi_{n+2,m+2}$$

where a_i is function of (n, m)

Shear couples a **finite number** of bases!!

Nonlinear Galaxy Properties

These nonlinear observables are defined with the linear observables.

Shear estimation with average ellipticity $\boxed{\langle w e_\alpha \rangle}$

(spin-0) selection weight and (spin-2) ellipticity

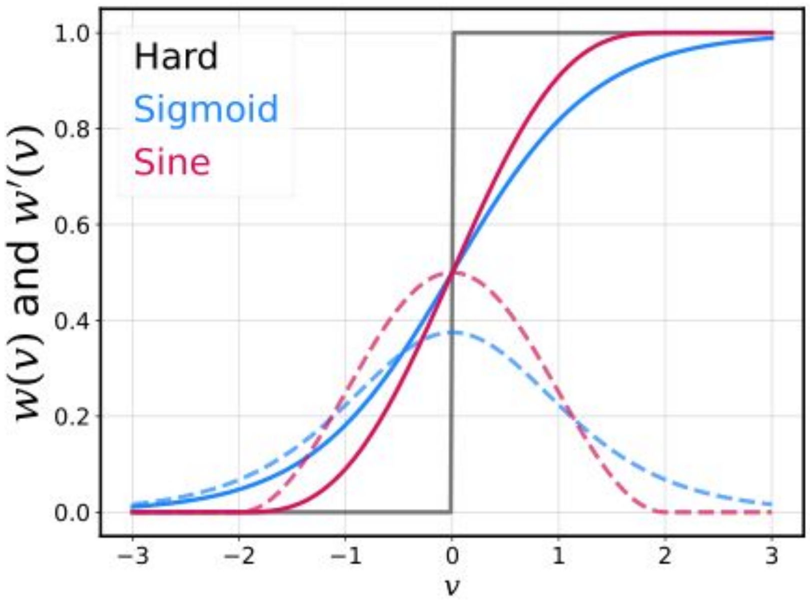
$$\boxed{w(\nu_i)} \quad \text{and} \quad \boxed{e_\alpha(\nu_i)}$$

$$w_2(\nu|\mu,\omega) = \begin{cases} 0 & \text{if } \nu \in (-\infty, \mu-\omega) \\ \frac{1}{2} + \frac{(\nu-\mu)}{2\omega} + \frac{1}{2\pi}\sin\left(\frac{(\nu-\mu)\pi}{\omega}\right) & \text{if } \nu \in [\mu-\omega, \mu+\omega] \\ 1 & \text{if } \nu \in (\mu+\omega, +\infty) \end{cases}$$

$$e_1 = \frac{M_{22c}}{M_{00}+\boxed{C}}, \quad e_2 = \frac{M_{22s}}{M_{00}+\boxed{C}}$$

C is a constant used to make sure the second-order noise perturbation is the leading order by setting e.g., C>STD(noise).

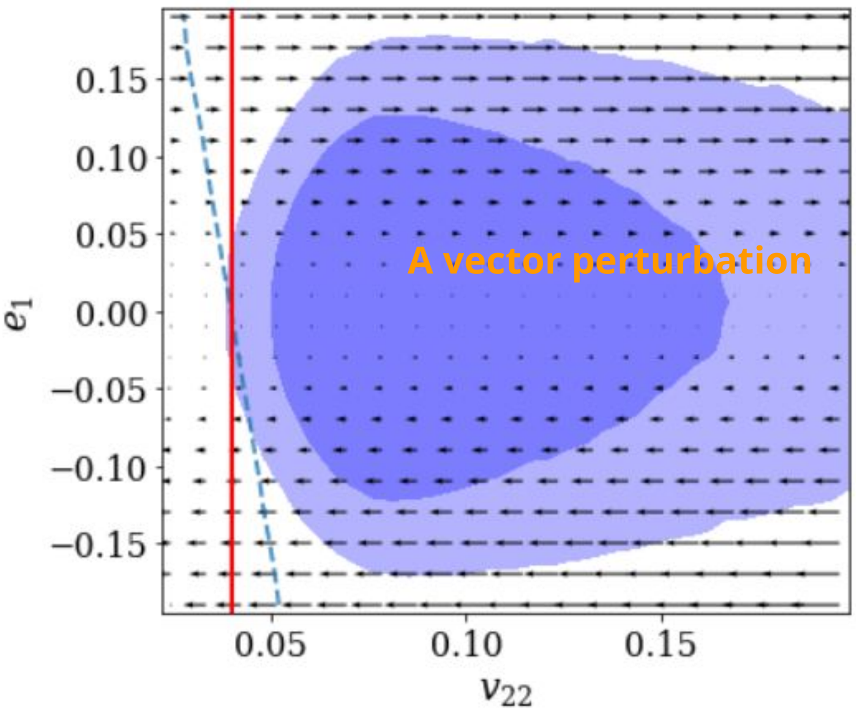
(see 1805.08514)



Shear is a Perturbation on Galaxy Number Distribution

$$\langle \boxed{w} e_\alpha \rangle = \langle \bar{w}\bar{e}_\alpha \rangle + \sum_{\beta=1,2} \boxed{\frac{\partial \langle w e_\alpha \rangle}{\partial g_\beta}} g_\beta + \mathcal{O}\left(g^3\right)$$

Selection weight



Shear response of average of weighted ellipticity: $\boxed{v_{i;\alpha} \equiv \partial v_i / \partial g_\alpha}$

$$\boxed{\mathcal{R}_\alpha} = \langle w e_{\alpha;\alpha} + w_{;\alpha} e_\alpha \rangle = \sum_i \left\langle w \frac{\partial e_\alpha}{\partial v_i} v_{i;\alpha} + e_\alpha \frac{\partial w}{\partial v_i} v_{i;\alpha} \right\rangle$$

<u>Difference to Metacal:</u> we interpret boundary as weight and derive how weight transforms under shear distortion.

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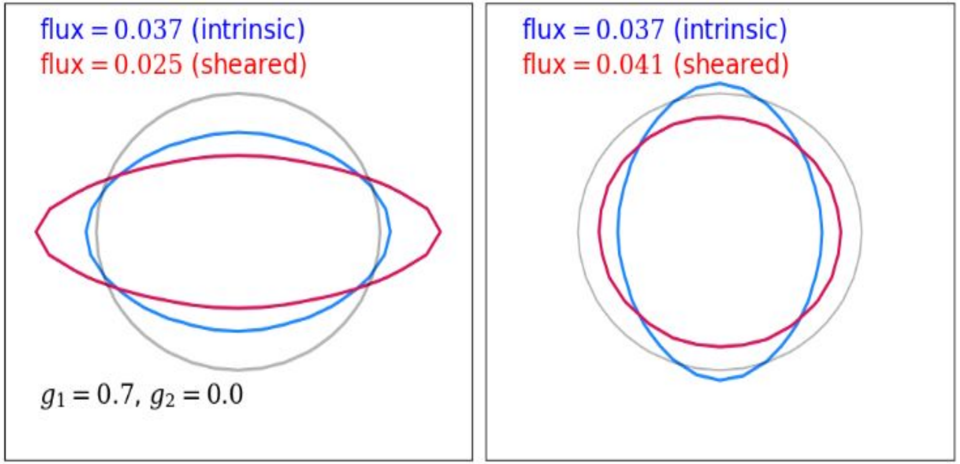
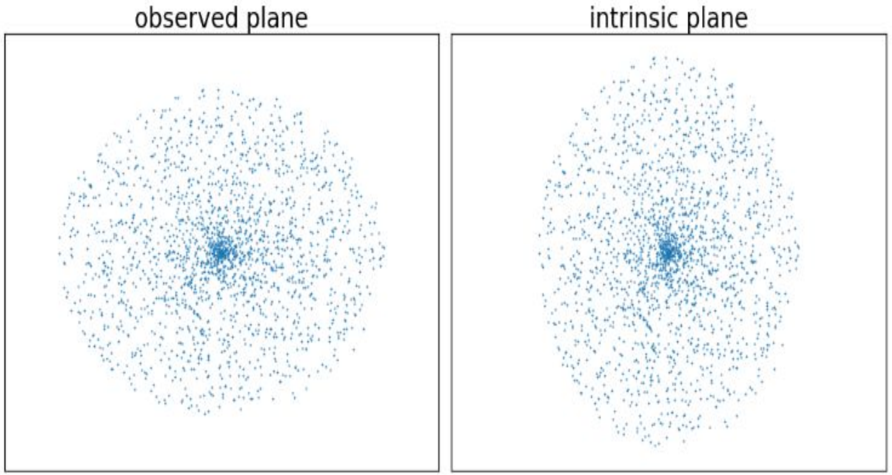
Shear Response of Detection matters

A galaxy is detected or not depends on shear:



For examples on the blended case, see Metadetection paper (2206.07683).

Where is the center of the measurement:



Isotropic offsets on the lensed plane are not isotropic on the intrinsic plane.

How to include detection into our formalism? — Two useful interpretations.

Smoothed Pixel Values are Projections onto Bases

$$\boxed{f^h_{\mathbf{x}}} = \frac{1}{(2\pi)^2} \iint \mathrm{d}^2 k\, f^h_{\mathbf{k}} e^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} = \frac{1}{(2\pi)^2} \iint \mathrm{d}^2 k\, f_{\mathbf{k}} \boxed{h_{\mathbf{k}} e^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}}$$

For one specific pixel at *x0*, its value is a projection of signal onto pixel basis in Fourier space:

$$f^h_{\mathbf{x}^0} = \iint \mathrm{d}^2 k\, f_{\mathbf{k}} \boxed{\left\{ \frac{1}{(2\pi)^2} \boxed{e^{-|\mathbf{k}|^2 \sigma_h^2/2}} e^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}^0} \right\}}$$

pixel basis:

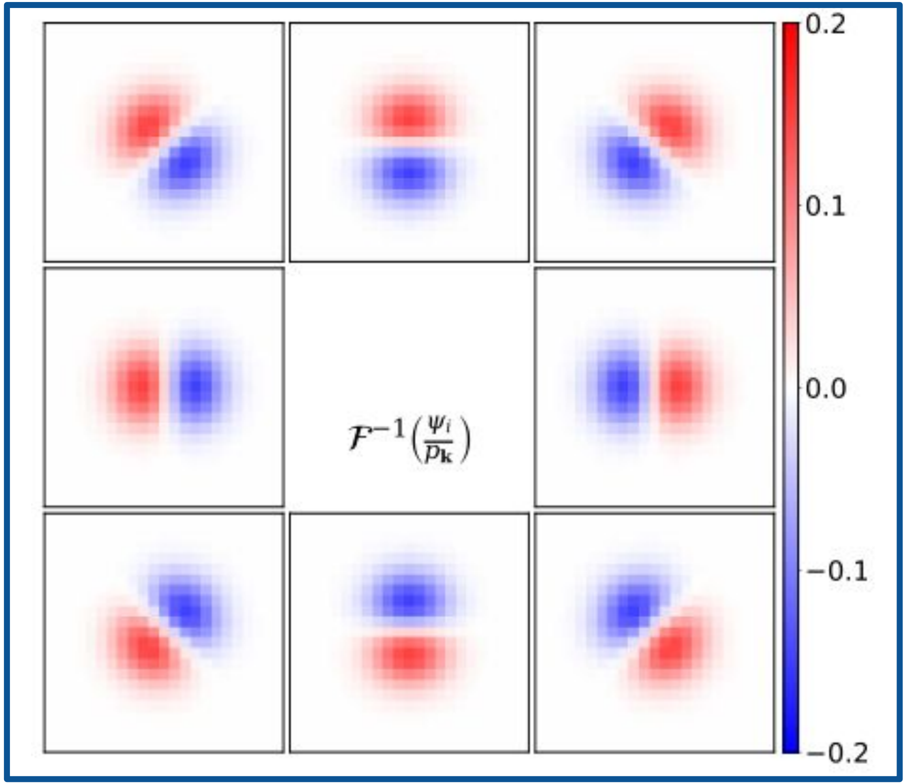
$$\phi^*_{\mathbf{x}^0}(\mathbf{k}) = \frac{1}{(2\pi)^2} e^{-|\mathbf{k}|^2 \sigma_h^2/2} e^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}^0}$$

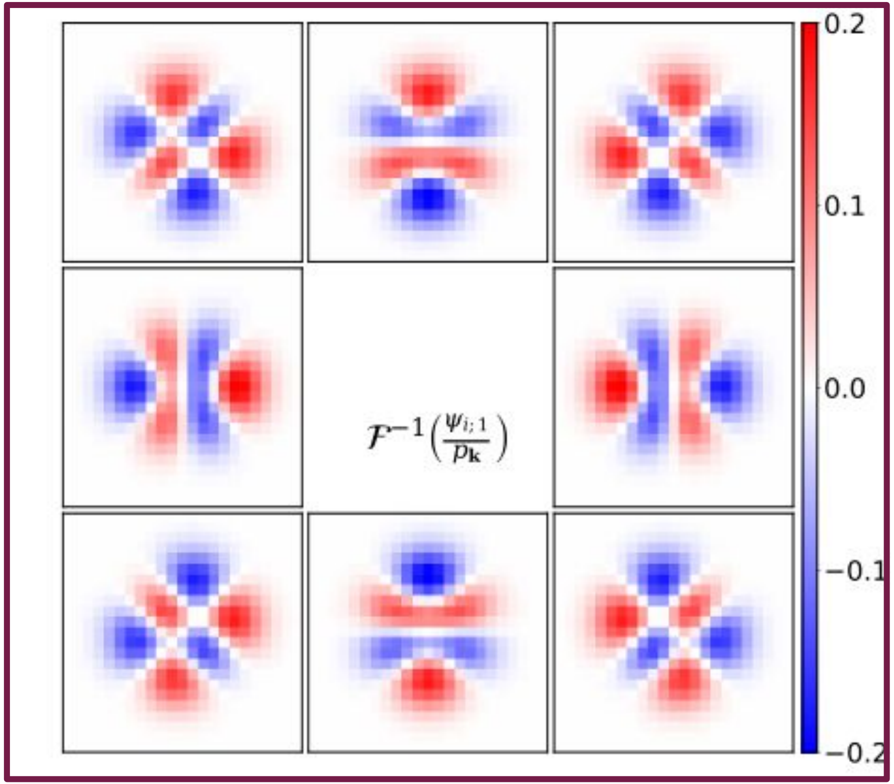
Shear response of pixel basis:

$$\frac{\partial \phi_{\mathbf{x}^0}}{\partial g_1} = \left((k_1^2 - k_2^2)\sigma_h^2 + \mathrm{i}x_1^0\, k_1 - \mathrm{i}x_2^0\, k_2 \right) \phi_{\mathbf{x}^0}$$

Peak Bases and their shear responses

$$\psi_i = \phi_{\mathbf{x}^0} - \phi_{\mathbf{x}^0 + (\cos(i\pi/4), \sin(i\pi/4))} \quad (i = 0 \ldots 7)$$





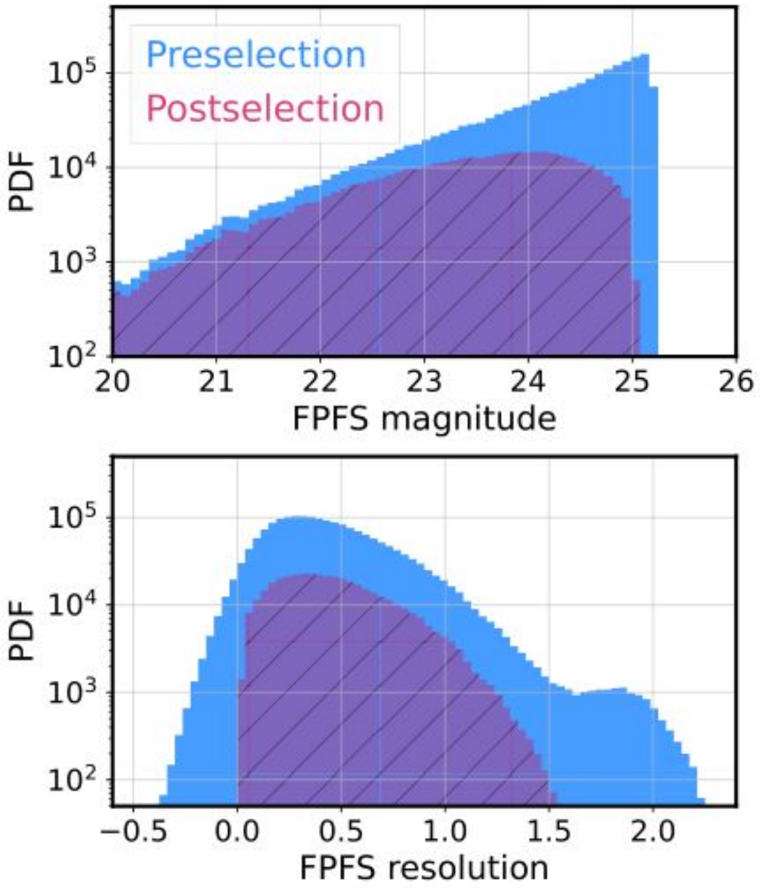
Detection is a Weighting by Peak modes

$$q_i = \iint \mathrm{d}^2 k \, \psi_i \, f_{\mathbf{k}}$$

$$q_{i;\alpha} = \iint \mathrm{d}^2 k \, \psi_{i;\alpha} \, f_{\mathbf{k}}$$

$$w(q_1)w(q_2)\ldots w(M_{00})\ldots$$

1. **Conduct a pre selection using 5 neighbouring pixels (using average PSF);**
2. **Conduct a post selection using all the peak modes with PSF models at each galaxy.**



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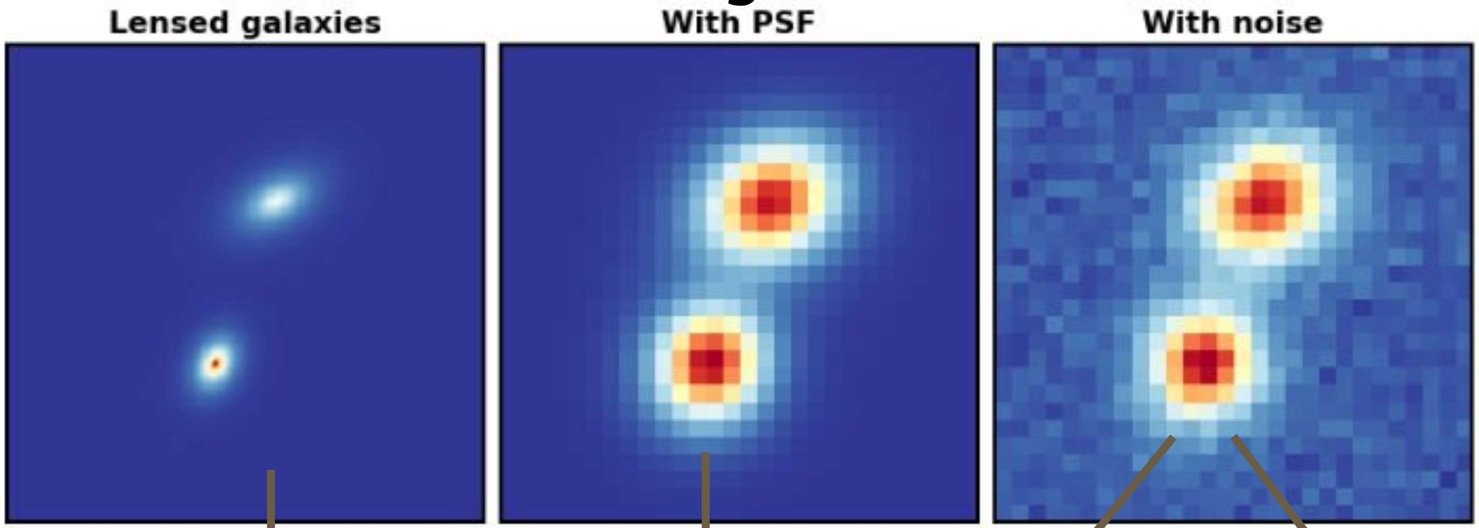
Systematics in weak lensing shear measurement



Lensed gals: $\nu\ ,\ e$

PSF smearing: $p_{\mathbf{x}}$

noise bias $\langle \tilde{e} \rangle \neq \langle e \rangle$

measurement error: $\delta\nu$, δe

$$\hat{g}_{1,2} = (1 + \boxed{m})g_{1,2} + \boxed{c_{1,2}}$$

multiplicative bias **additive bias**

PSF revision: Deconvolution in Fourier space

$$f^p(\mathbf{x}) = \int f(\mathbf{x}) p(\mathbf{x} - \mathbf{x}') \mathrm{d}^2 x'$$

$$f_{\mathbf{k}} = \frac{f^p_{\mathbf{k}}}{\boxed{p_{\mathbf{k}}}}$$

Fourier transform of PSF

Then project on bases (i.e., shapelet and peak bases):

$$M_{nm} = \int \chi^*_{nm} \frac{f^p_{\mathbf{k}}}{p_{\mathbf{k}}} \mathrm{d}^2 k$$

Note: scale radius of the basis functions should be larger than the scale radius of PSF in configuration space (equivalently, smaller in Fourier space) to avoid noise on small scales being boosted by PSF deconvolution.

Noise is a Perturbation on Galaxy Number Distribution

Noise bias correction:

$$\langle \boxed{\tilde{w}\tilde{e}_\alpha} \rangle = \langle w e_\alpha \rangle + \frac{1}{2}\sum_{i,j} \left\langle \boxed{\frac{\partial^2 (\tilde{w}\tilde{e}_\alpha)}{\partial v_i \partial v_j}} \boxed{\delta v_i \delta v_j} \right\rangle + \mathcal{O}\left((\delta v_i)^4\right)$$

Noisy observables

Independent of gals

The leading order perturbation is a contraction between Hessian matrix (noise response) and covariance matrix of measurement error on the linear observables.

Under what condition the analytic covariance of measurement errors from image noise can be derived:

	not correlated	correlated
Homogeneous	✅	✅
inhomogeneous	✅	❌

(correlated and homogeneous are defined in configuration space.)

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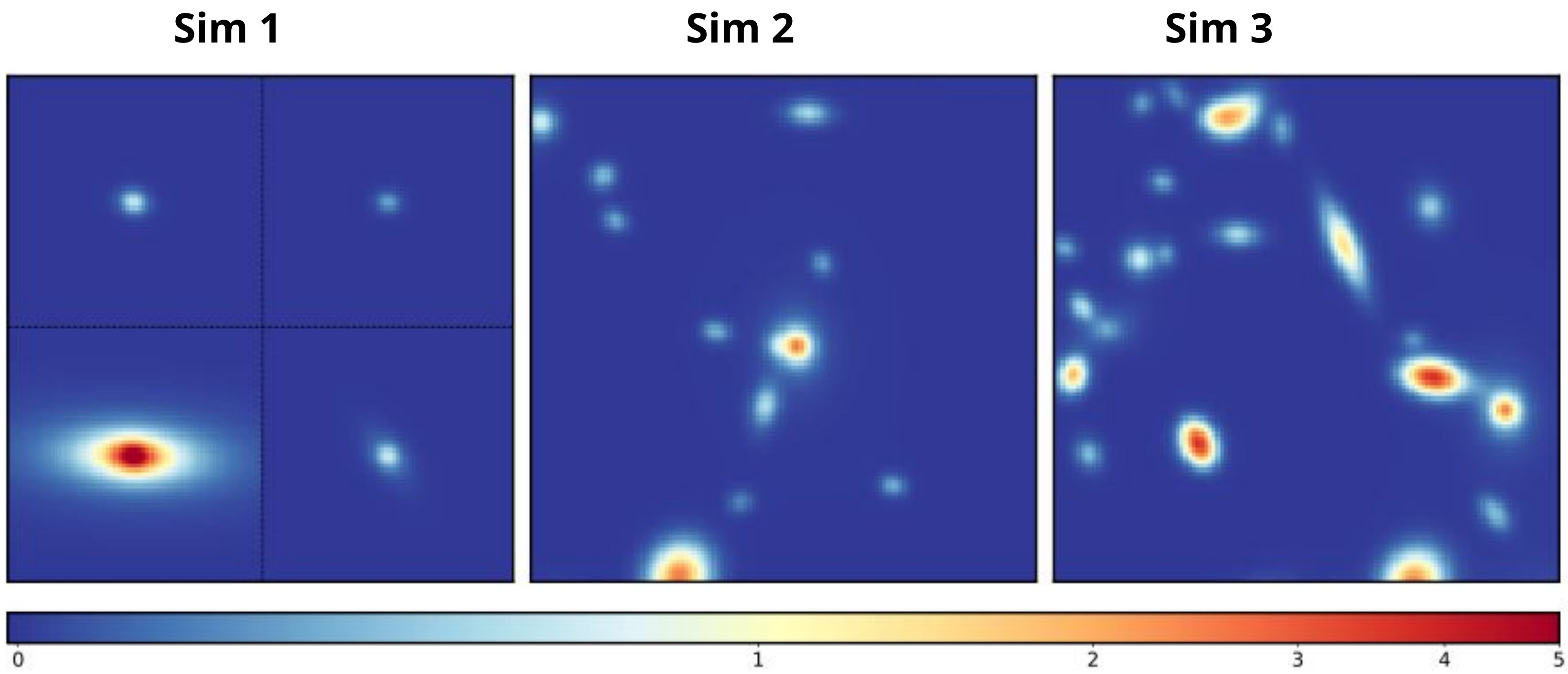


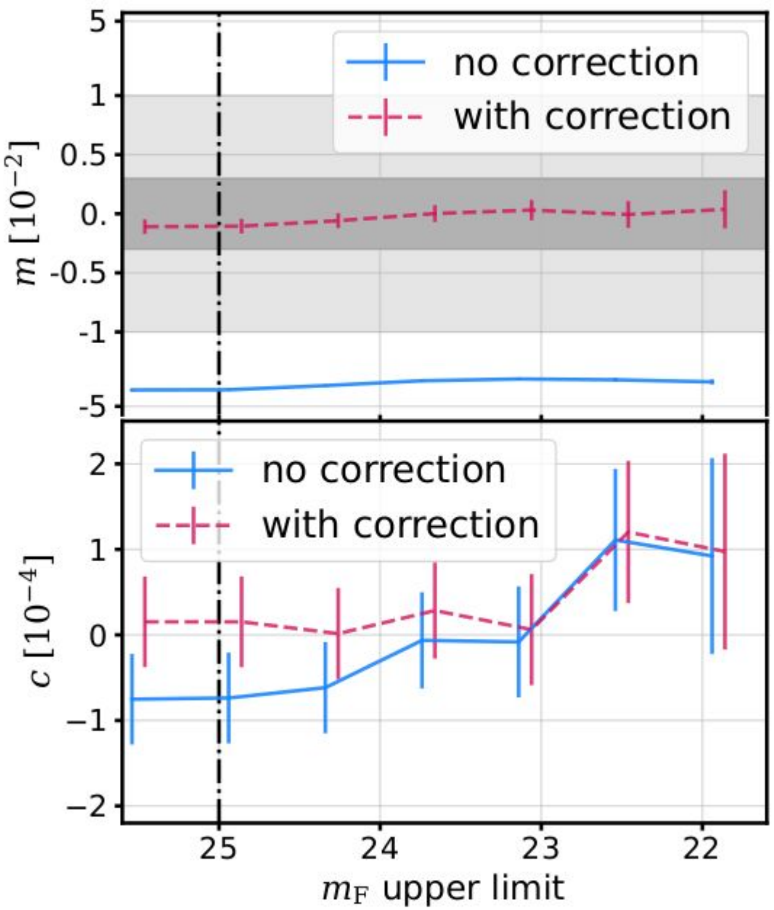
Constant-shear Simulations

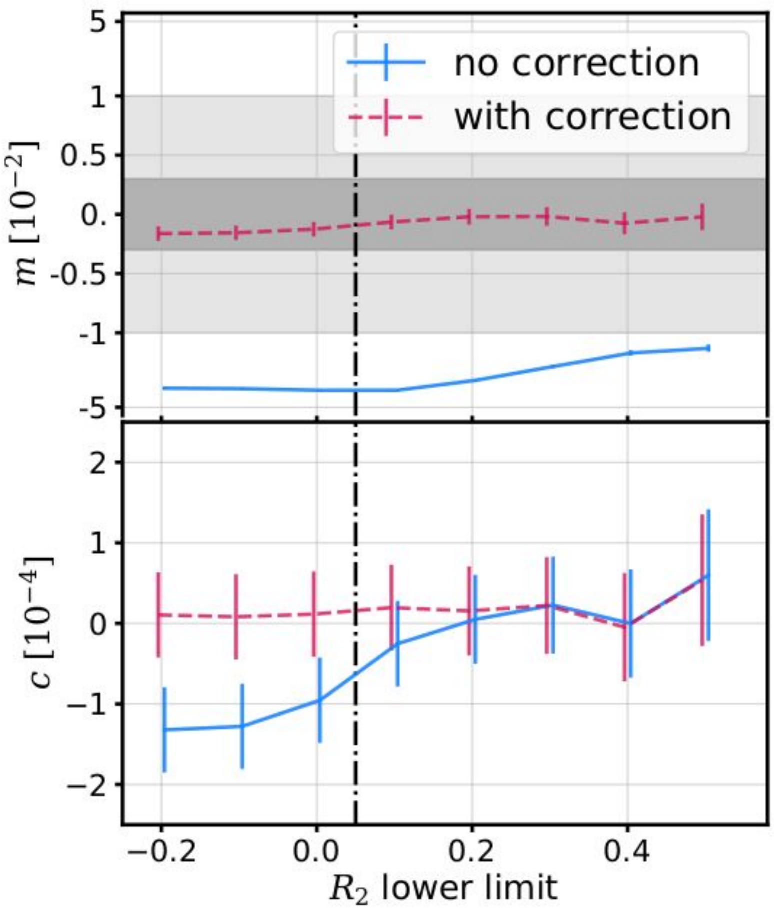
Figure 6. The left panel shows a 128 pix × 128 pix (or, equivalently, 0.36×0.36 arcmin^2) region of the isolated stamp-based galaxy image simulation, where the black lines indicate the boundaries of the 64 pix × 64 pix stamps. The middle and the right panels are for blended galaxy image simulations with input densities of 85 arcmin^{-2} and 170 arcmin^{-2}, respectively. The images shown here are noiseless for demonstration, but different noise realizations are added to the galaxy images during the tests of shear estimation.

We run detection and shear estimator on these three sets of image simulations.

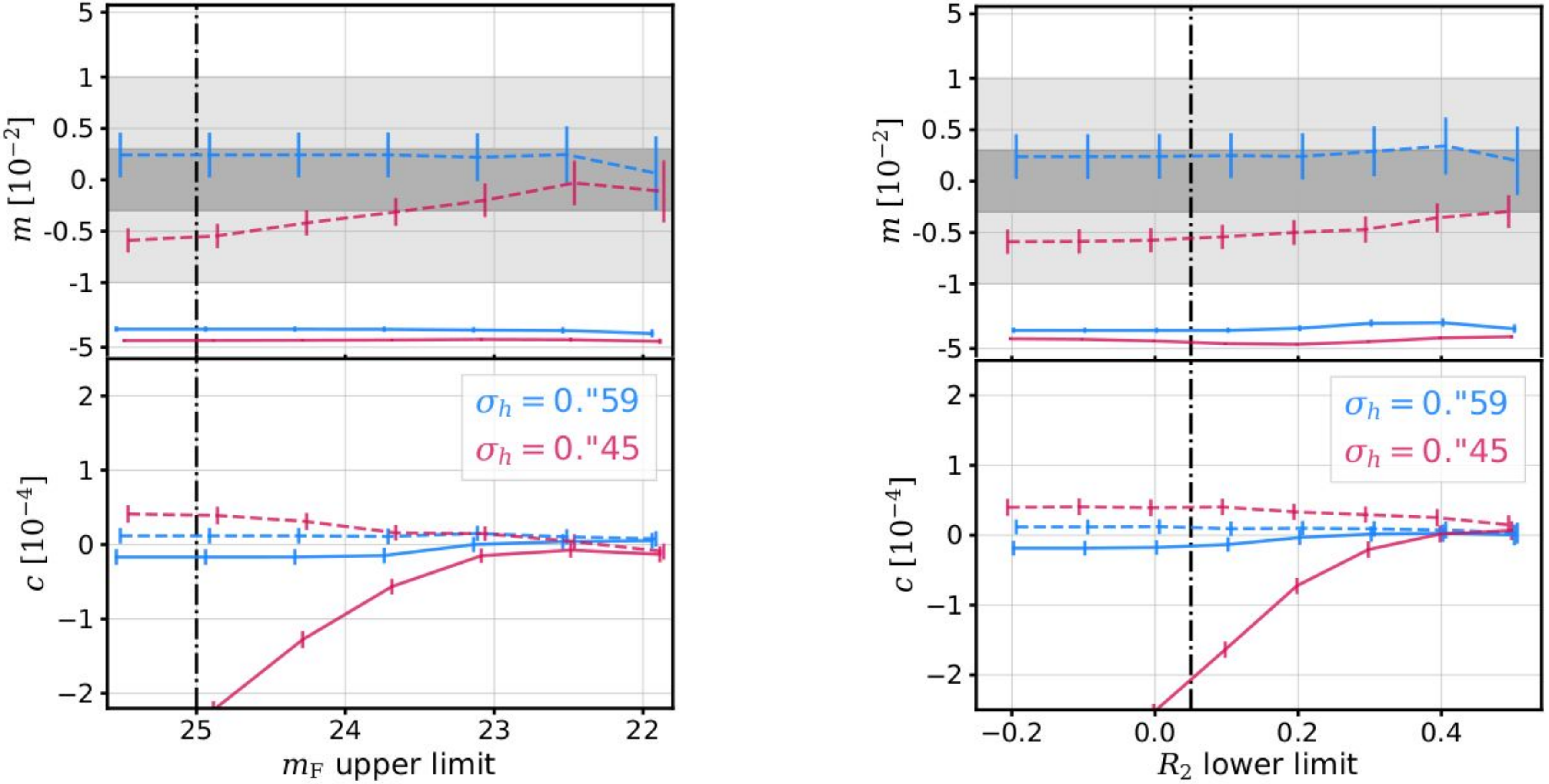
Sim 1:

dashed / solid: with / without analytic bias correction

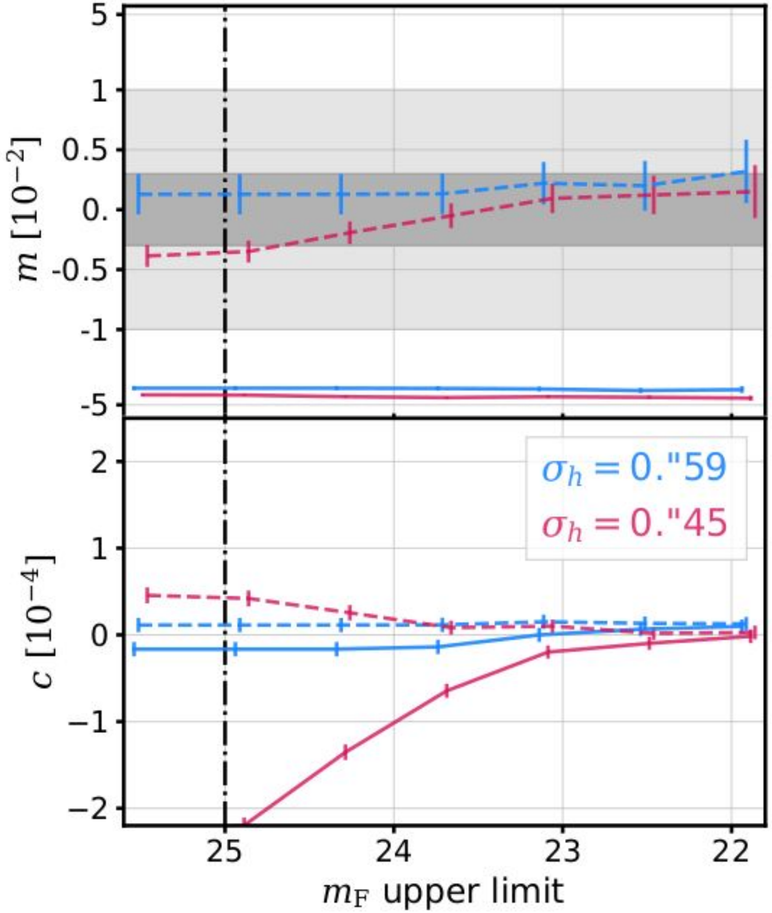


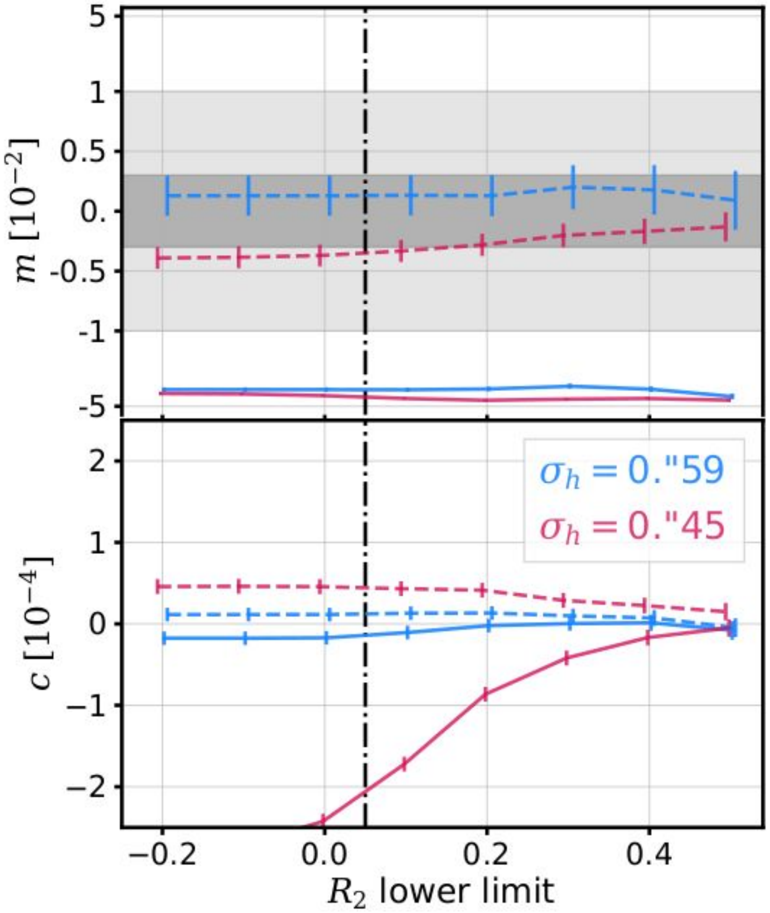


Sim 2: dashed / solid: with / without analytic bias correction



Sim 3: dashed / solid: with / without analytic bias correction





Summary

1. The analytical shear estimator can control the multiplicative bias **below 1%** even for **blended galaxies**;

2. The analytical shear estimator can process **~1000 galaxies** per cpu second;

3. The analytical shear estimator does not needs creation of multiple catalogs, and is very simple to implement.

Thanks!

Codes are available at https://github.com/mr-superonion/FPFS

Reference

The following papers are ready to be cited if you find any of these papers interesting or use the pipeline. Comments are welcome.

- version 3: Li & Mandelbaum (2022) correct for detection bias from pixel level by interpreting smoothed pixel values as a projection of signal onto a set of basis functions.
- version 2: Li , Li & Massey (2022) derive the covariance matrix of FPFS measurements and corrects for noise bias to second-order. In addition, it derives the correction for selection bias.
- version 1: Li et. al (2018) build up the FPFS formalism based on Fourier_Quad and polar shapelets.

Backups: The weighting parameter *C*