

***Analytical* Shear Responses of Galaxy properties and Detection**

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Based on Li et. al (2018); Li, Li & Massey (2022);
Li & Mandelbaum (2022)

Collaborators: Rachel Mandelbaum, Richard Massey,
Surhud More,..++++

1. **Shear Responses of Galaxy properties**
2. Shear Response of Detection
3. PSF and Noise Corrections
4. Tests on Image Simulations
5. Future Tests

Measuring Shear Distortion

intrinsic
gal

$$\bar{f}(\bar{\mathbf{x}})$$

$$= f(\mathbf{x})$$

lensed
gal

$$\bar{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

$$\mathbf{A} = \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}$$

where $g_{1,2} \ll 1$

Develop an accurate analytic shear estimator even for blended gals after detection/selection.

Assumptions:

- (i) intrinsic galaxies are randomly oriented;
- (ii) image noise is symmetric to zero.

	< 0	> 0
κ		
g_1		
g_2		

Metacalibration to estimate linear shear response

We begin from linear observable and will generalize to non-linear observables.

Observable ν : projection of galaxy onto a basis —

$$\begin{aligned}\nu &= \int f(\mathbf{x})\chi(\mathbf{x})d^2x \\ &= \int \bar{f}(\mathbf{A}\mathbf{x})\chi(\mathbf{x})d^2x\end{aligned}$$

How does ν transform under a shear distortion?

**Huff & Mandelbaum (2017),
Sheldon & Huff (2017):**

$$\begin{aligned}\nu &= \bar{\nu} + g_1 \boxed{\frac{\partial \nu}{\partial g_1}} + g_2 \boxed{\frac{\partial \nu}{\partial g_2}} \\ \boxed{\frac{\partial \nu}{\partial g_i}} &= \frac{\partial (\int \bar{f}(\mathbf{A}\mathbf{x})\chi(\mathbf{x})d^2x)}{\partial g_i}\end{aligned}$$

Shear response can be estimated by adding small shear distortions to galaxy image.

A **passive** line of thought: distortions on bases

$$\vec{x} \rightarrow \mathbf{J}\vec{x}$$

$$\mathbf{J} = \mathbf{A}^{-1}$$

$$O = \int \bar{f}(\mathbf{A}\mathbf{x}) \chi(\mathbf{x}) d^2 x$$

$$O \propto \int \bar{f}(\mathbf{x}) \chi(\mathbf{J}\mathbf{x}) d^2 x$$

If we **fix the galaxy images** but **distort the bases**, the measurement is **faster**, since we can pre-derive (compute) the **shear responses of the bases**.

$$\frac{\partial O}{\partial g_i} \propto \int \bar{f}(\mathbf{x}) \frac{\partial \chi(\mathbf{J}\mathbf{x})}{\partial g_i} d^2 x$$

Chain rule for derivatives

$$\begin{aligned} \frac{\partial \chi(\mathbf{J}\mathbf{x})}{\partial g_1} &= \left(x \frac{\partial}{\partial x} - y \frac{\partial}{\partial y} \right) \chi(\mathbf{x}) \\ \frac{\partial \chi(\mathbf{J}\mathbf{x})}{\partial g_2} &= \left(x \frac{\partial}{\partial y} + y \frac{\partial}{\partial x} \right) \chi(\mathbf{x}) \end{aligned}$$

Shaplet Bases: Eigen functions of 2D QHO

Massey & Refregier (2005):

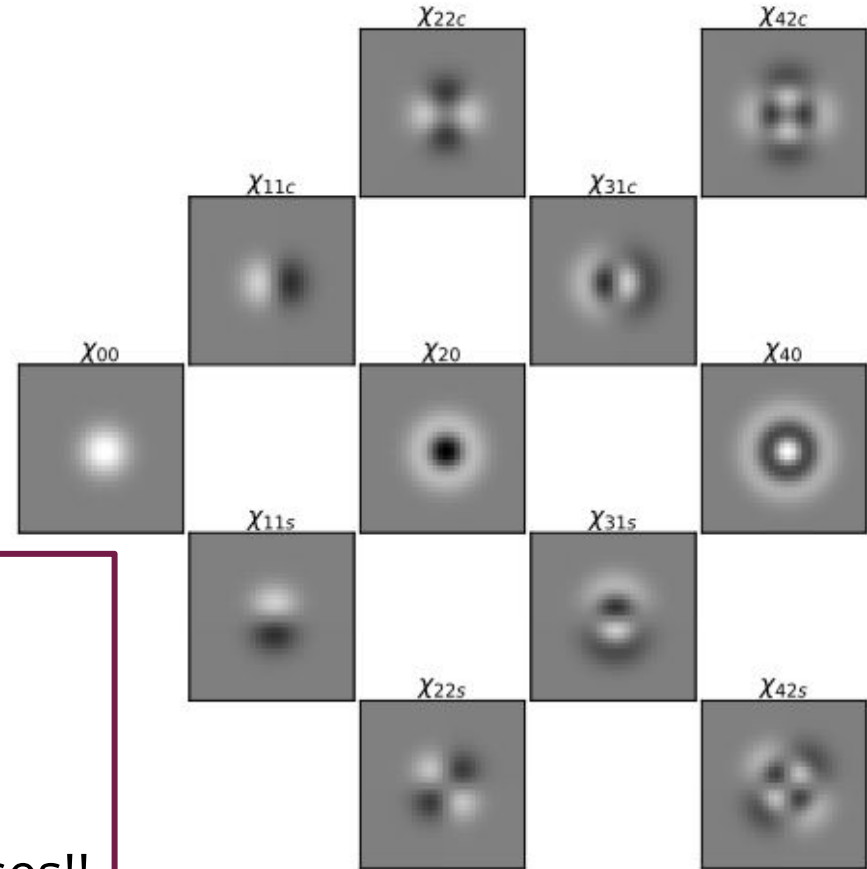
$$M_{nm} = \int f(\mathbf{x}) \chi_{nm} d^2x$$

- 'n' determines **radial profile**;
- 'm' determines **spin**;
- 'c' refers to **real part** (cosine);
- 's' refers to **imaginary part** (sine).

$$\frac{\partial \chi_{n,m}(\mathbf{Jx})}{\partial g_1} = a_1 \chi_{n-2,m-2} + a_2 \chi_{n+2,m-2} \\ + a_3 \chi_{n-2,m+2} + a_4 \chi_{n+2,m+2}$$

where a_i is function of (n, m)

Shear couples a **finite number** of bases!!



Nonlinear Galaxy Properties

Shear estimation with **average ellipticity** $\langle we_\alpha \rangle$
 (spin-0) **selection weight** and (spin-2) **ellipticity**

$$w(\nu_i)$$

and

$$e_\alpha(\nu_i)$$

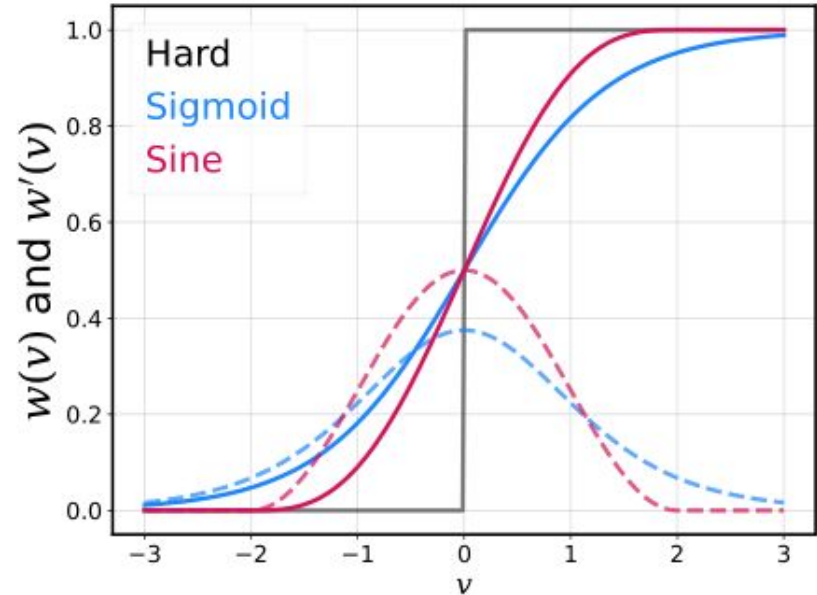
$$e_1 = \frac{M_{22c}}{M_{00} + C}, \quad e_2 = \frac{M_{22s}}{M_{00} + C}$$

C is a constant used to make sure the second-order noise perturbation is the leading order by setting e.g., **C** > STD(noise).

(see 1805.08514)

These nonlinear observables are defined with the linear observables.

$$w_2(v|\mu, \omega) = \begin{cases} 0 & \text{if } v \in (-\infty, \mu - \omega) \\ \frac{1}{2} + \frac{(v-\mu)}{2\omega} + \frac{1}{2\pi} \sin\left(\frac{(v-\mu)\pi}{\omega}\right) & \text{if } v \in [\mu - \omega, \mu + \omega] \\ 1 & \text{if } v \in (\mu + \omega, +\infty) \end{cases}$$



Shear is a Perturbation on Galaxy Number Distribution

$$\langle \boxed{w} e_\alpha \rangle = \langle \bar{w} \bar{e}_\alpha \rangle$$

Selection weight

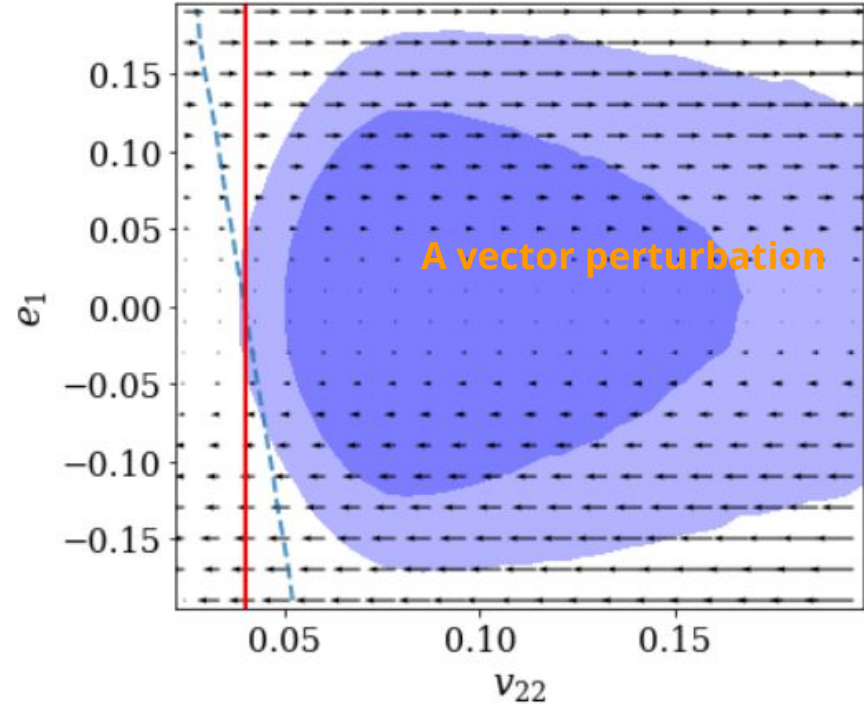
$$+ \sum_{\beta=1,2} \boxed{\frac{\partial \langle w e_\alpha \rangle}{\partial g_\beta}} g_\beta + \mathcal{O}(g^3)$$

Shear response of average of **weighted** ellipticity:

$$\boxed{v_{i;\alpha} \equiv \partial v_i / \partial g_\alpha}$$

$$\boxed{\mathcal{R}_\alpha} = \langle w e_{\alpha;\alpha} + w_{;\alpha} e_\alpha \rangle$$

$$= \sum_i \left\langle w \frac{\partial e_\alpha}{\partial v_i} v_{i;\alpha} + e_\alpha \frac{\partial w}{\partial v_i} v_{i;\alpha} \right\rangle$$



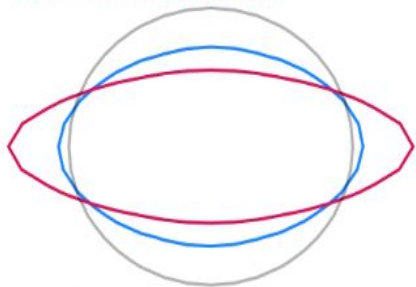
Difference to Metacal: we interpret **boundary** as weight and derive how **weight** transforms under shear distortion.

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Shear Response of Detection matters

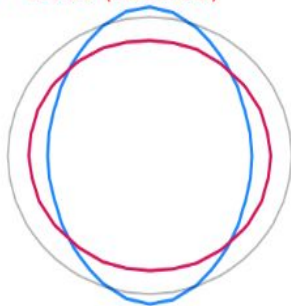
A galaxy is **detected or not** depends on shear:

flux = 0.037 (intrinsic)
flux = 0.025 (sheared)



$g_1 = 0.7, g_2 = 0.0$

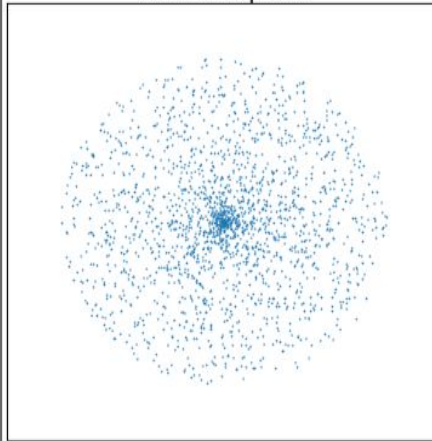
flux = 0.037 (intrinsic)
flux = 0.041 (sheared)



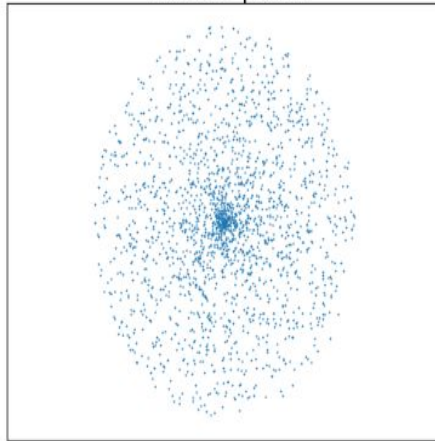
For examples on the blended case, see
Metadetection paper (2206.07683).

Where is the **center** of the measurement:

observed plane



intrinsic plane



Isotropic offsets on the lensed plane are
not isotropic on the intrinsic plane.

How to include detection into our formalism? — Two useful interpretations.

Smoothed Pixel Values are Projections onto Bases

$$\boxed{f_{\mathbf{x}}^h} = \frac{1}{(2\pi)^2} \iint d^2 k f_{\mathbf{k}}^h e^{i\mathbf{k} \cdot \mathbf{x}} = \frac{1}{(2\pi)^2} \iint d^2 k f_{\mathbf{k}} \boxed{h_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{x}}}$$

For one specific pixel at $\mathbf{x}_0$, its value is a projection of signal onto **pixel basis** in Fourier space:

$$f_{\mathbf{x}_0}^h = \iint d^2 k f_{\mathbf{k}} \left\{ \frac{1}{(2\pi)^2} \boxed{e^{-|\mathbf{k}|^2 \sigma_h^2 / 2}} e^{i\mathbf{k} \cdot \mathbf{x}_0} \right\}$$

pixel basis:

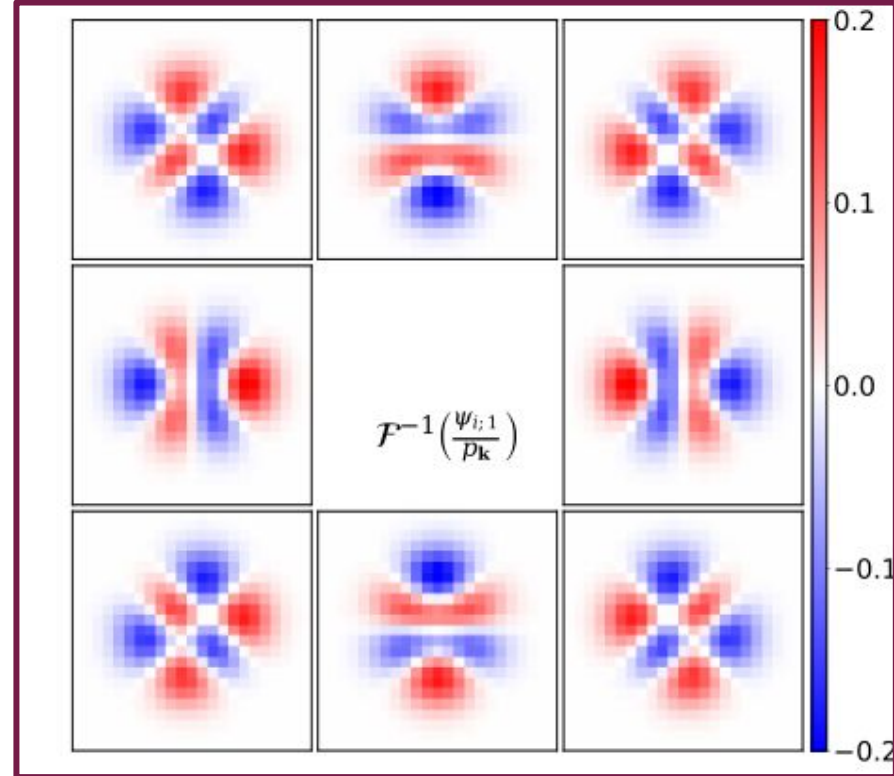
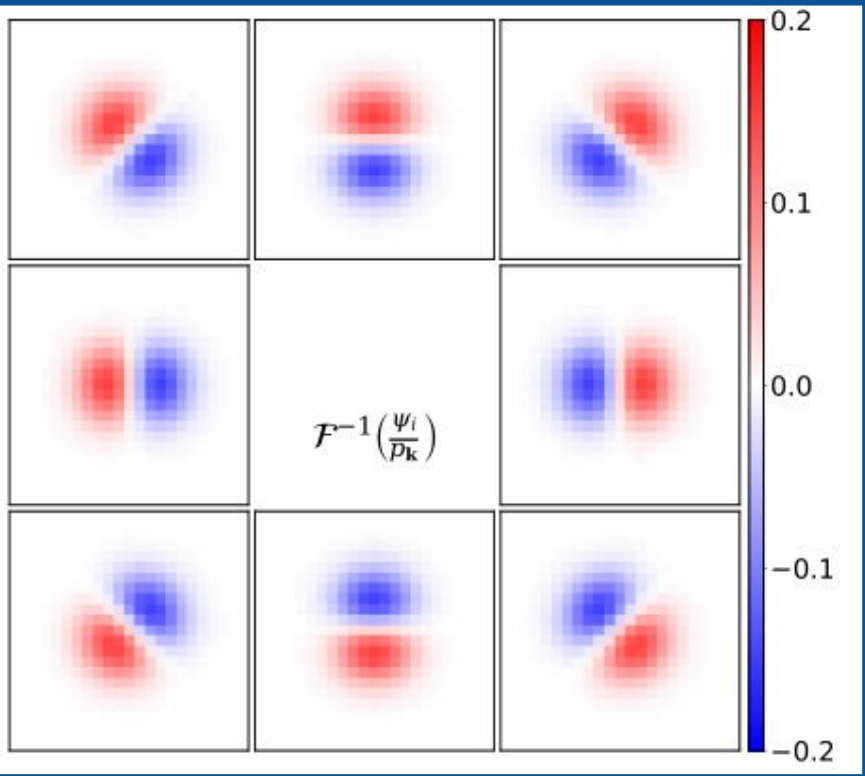
$$\phi_{\mathbf{x}_0}^*(\mathbf{k}) = \frac{1}{(2\pi)^2} e^{-|\mathbf{k}|^2 \sigma_h^2 / 2} e^{i\mathbf{k} \cdot \mathbf{x}_0}$$

Shear response of pixel basis:

$$\frac{\partial \phi_{\mathbf{x}_0}}{\partial g_1} = ((k_1^2 - k_2^2) \sigma_h^2 + i x_1^0 k_1 - i x_2^0 k_2) \phi_{\mathbf{x}_0}$$

Peak Bases and their shear responses

$$\psi_i = \phi_{\mathbf{x}^0} - \phi_{\mathbf{x}^0 + (\cos(i\pi/4), \sin(i\pi/4))}$$



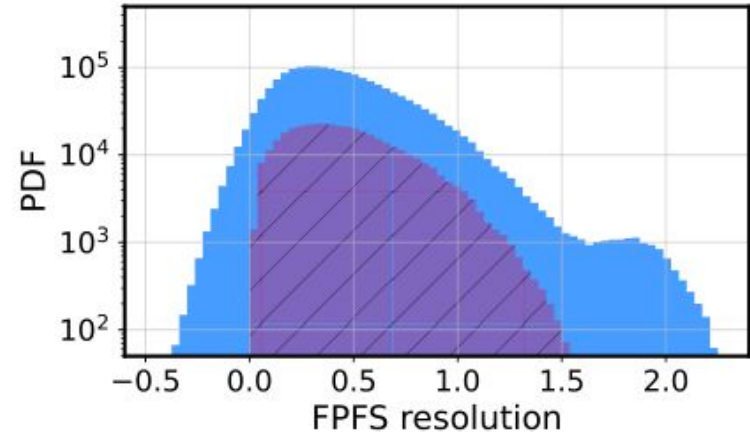
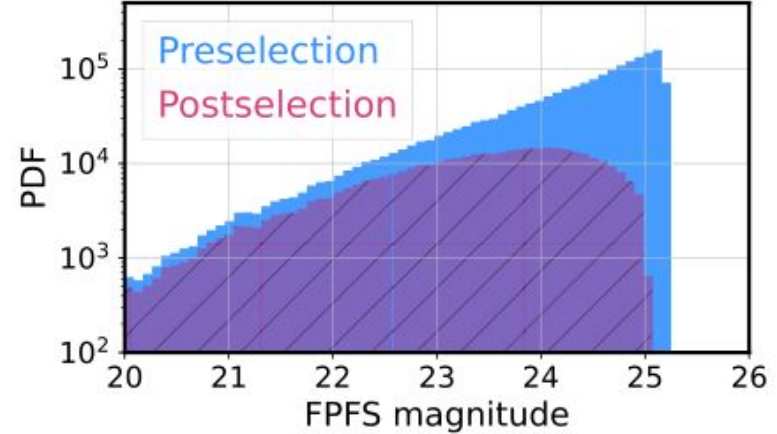
Detection is a Weighting by Peak modes

$$q_i = \iint d^2 k \psi_i f_{\mathbf{k}}$$

$$q_{i;\alpha} = \iint d^2 k \psi_{i;\alpha} f_{\mathbf{k}}$$

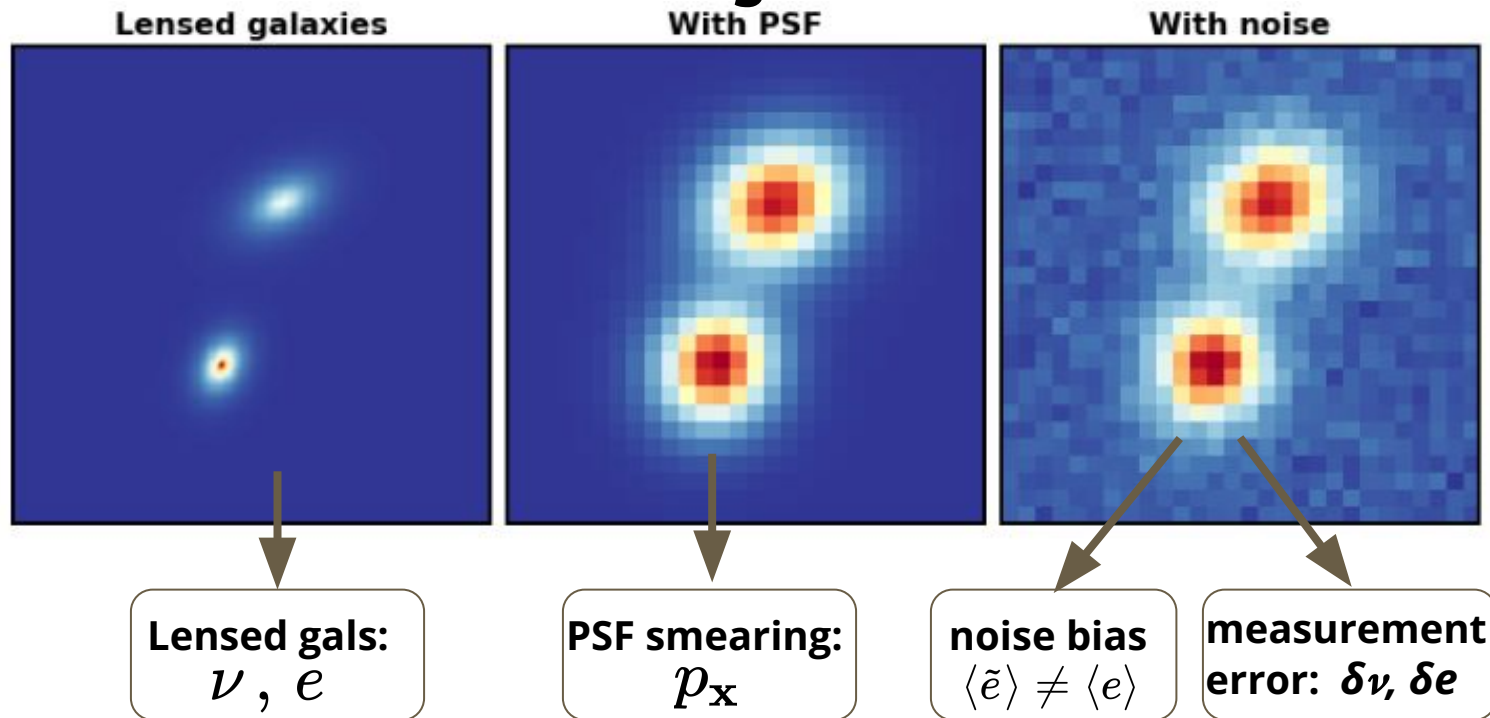
$$w(q_1, q_2, \dots, M_{00}, M_{20})$$

1. Conduct a pre selection using 5 neighbouring pixels (using average PSF);
2. Conduct a post selection using all the peak modes with PSF models at each galaxy.



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Systematics in weak lensing shear measurement



$$\hat{g}_{1,2} = (1 + \boxed{m})g_{1,2} + \boxed{c_{1,2}}$$

multiplicative bias

additive bias

PSF revision: **Deconvolution** in Fourier space

$$f^p(\mathbf{x}) = \int f(\mathbf{x})p(\mathbf{x} - \mathbf{x}')d^2x'$$

$$f_{\mathbf{k}} = \frac{f_{\mathbf{k}}^p}{p_{\mathbf{k}}} \rightarrow \text{Fourier transform of PSF}$$

Then project on bases (i.e., shapelet and peak bases):

$$M_{nm} = \int \chi_{nm}^* \frac{f_{\mathbf{k}}^p}{p_{\mathbf{k}}} d^2k$$

Note: scale radius of the basis functions should be larger than the scale radius of PSF in configuration space (equivalently, smaller in Fourier space) to avoid noise on small scales being boosted by PSF deconvolution.

Noise is a Perturbation on Galaxy Number Distribution

Noise bias correction:

$$\langle \tilde{w} \tilde{e}_\alpha \rangle = \langle w e_\alpha \rangle + \frac{1}{2} \sum_{i,j} \left\langle \frac{\partial^2 (\tilde{w} \tilde{e}_\alpha)}{\partial v_i \partial v_j} \delta v_i \delta v_j \right\rangle + \mathcal{O}((\delta v_i)^4)$$

Noisy observables

Independent of gals

The leading order perturbation is a contraction between **Hessian matrix** (noise response) and **covariance matrix** of measurement error on the linear observables.

Under what condition the **analytic covariance** of measurement errors from image noise can be derived:

	not correlated	correlated
Homogeneous	✓	✓
inhomogeneous	✓	✗

(correlated and homogeneous are defined in configuration space.)

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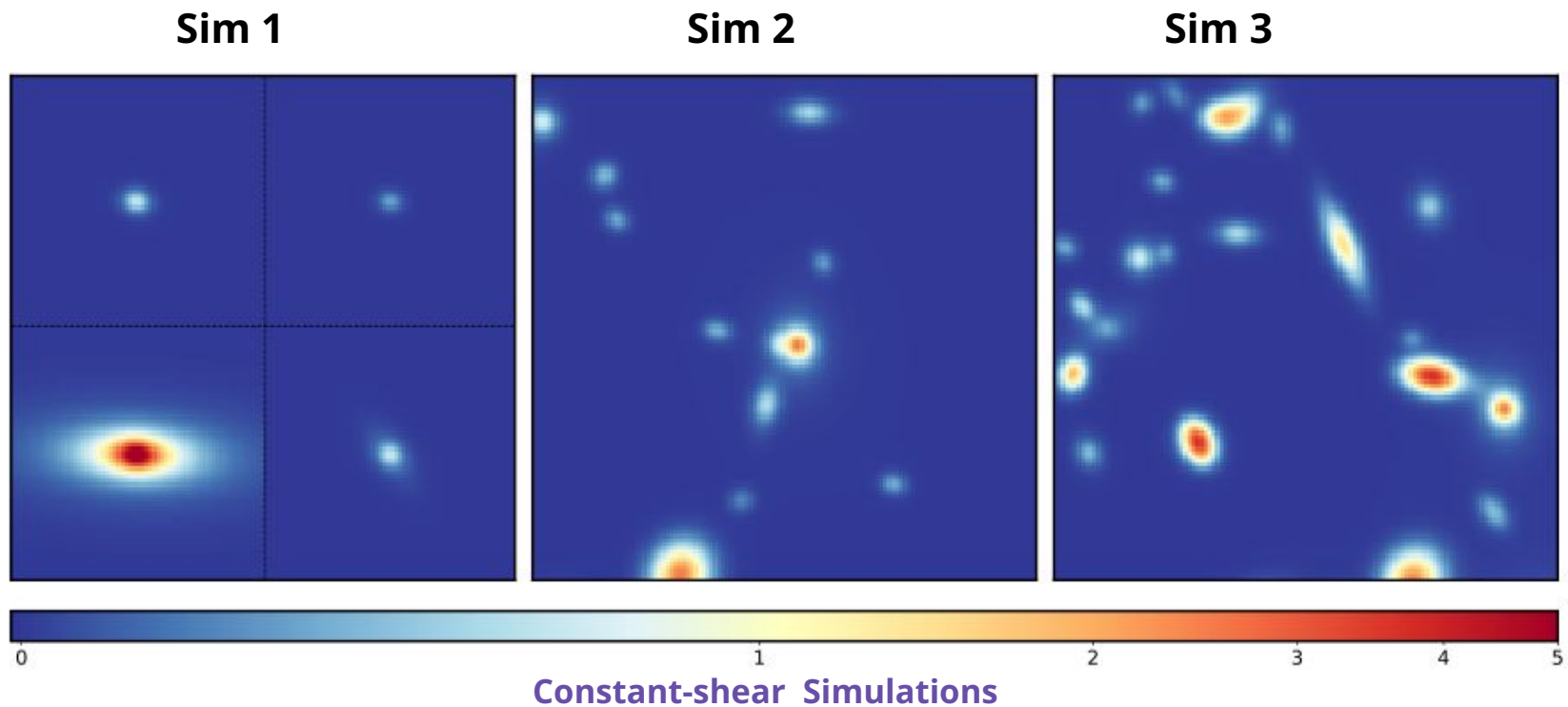
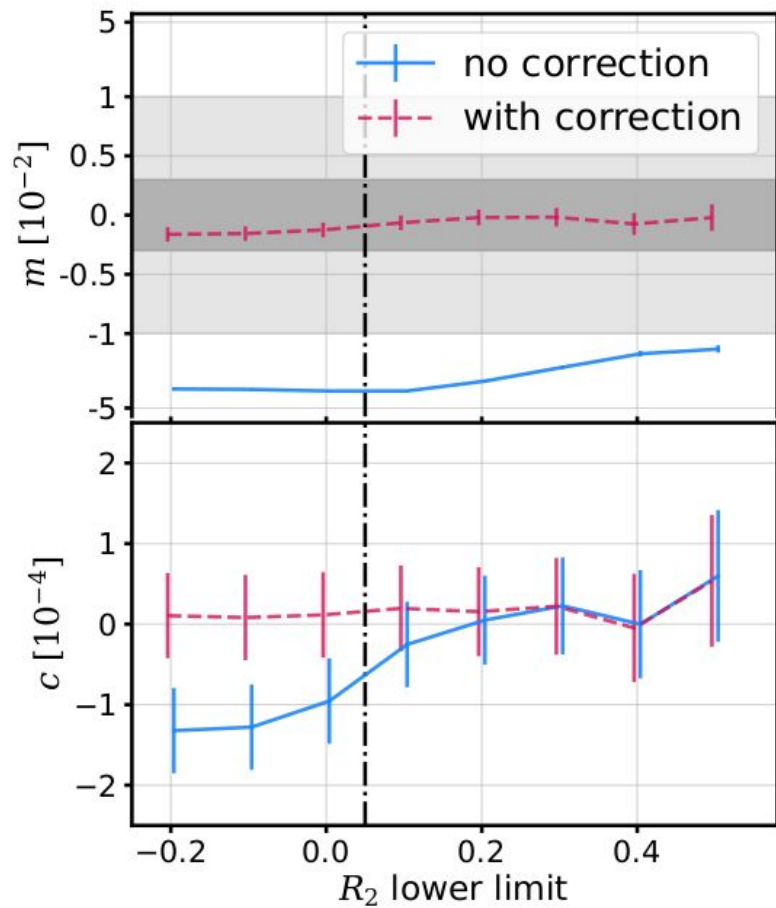
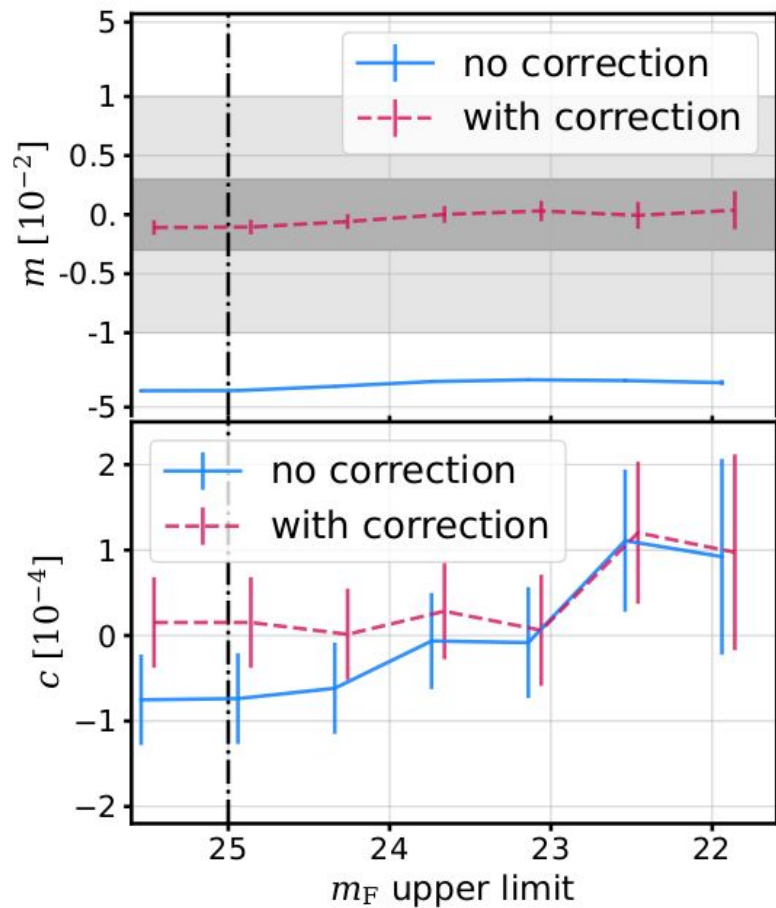


Figure 6. The left panel shows a $128 \text{ pix} \times 128 \text{ pix}$ (or, equivalently, $0.36 \times 0.36 \text{ arcmin}^2$) region of the isolated stamp-based galaxy image simulation, where the black lines indicate the boundaries of the $64 \text{ pix} \times 64 \text{ pix}$ stamps. The middle and the right panels are for blended galaxy image simulations with input densities of 85 arcmin^{-2} and 170 arcmin^{-2} , respectively. The images shown here are noiseless for demonstration, but different noise realizations are added to the galaxy images during the tests of shear estimation.

We run detection and shear estimator on these three sets of image simulations.

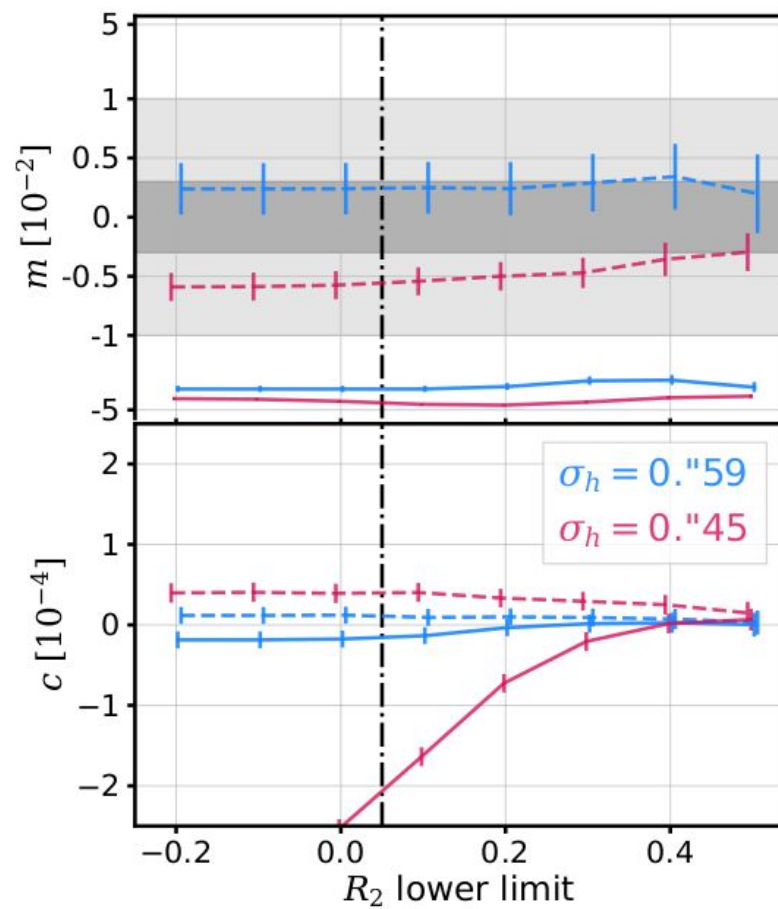
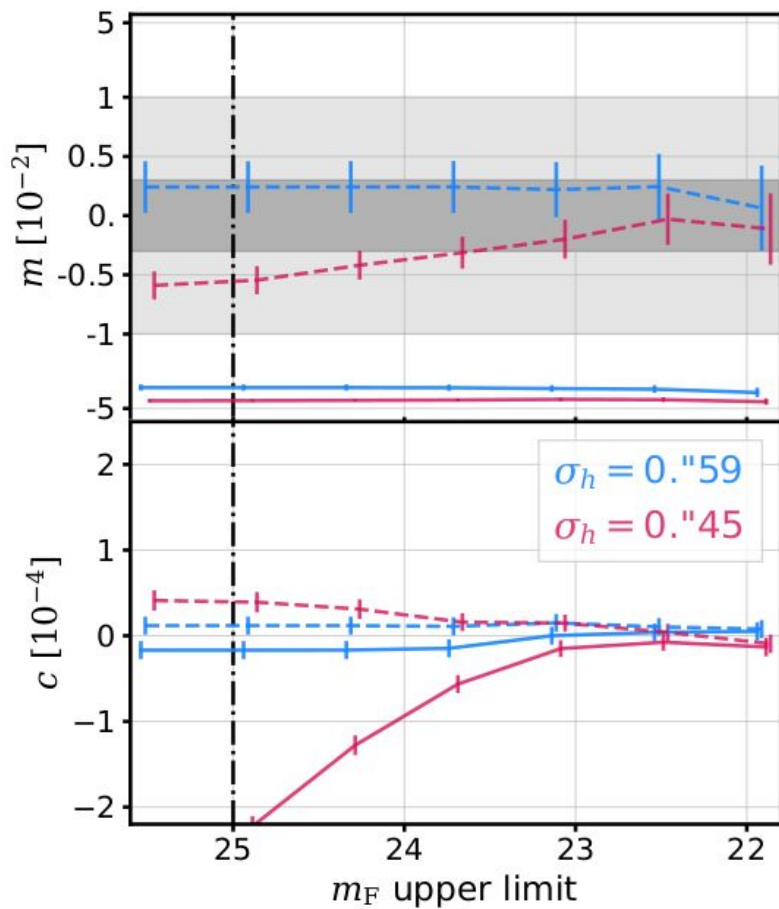
Sim 1:

dashed / solid: with / without analytic bias correction

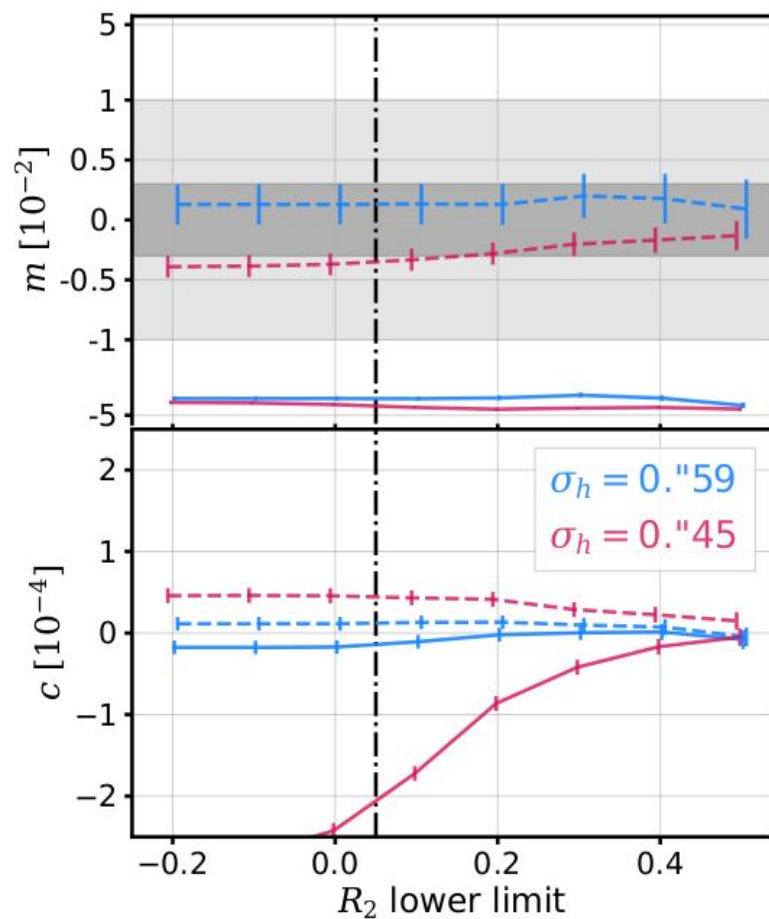
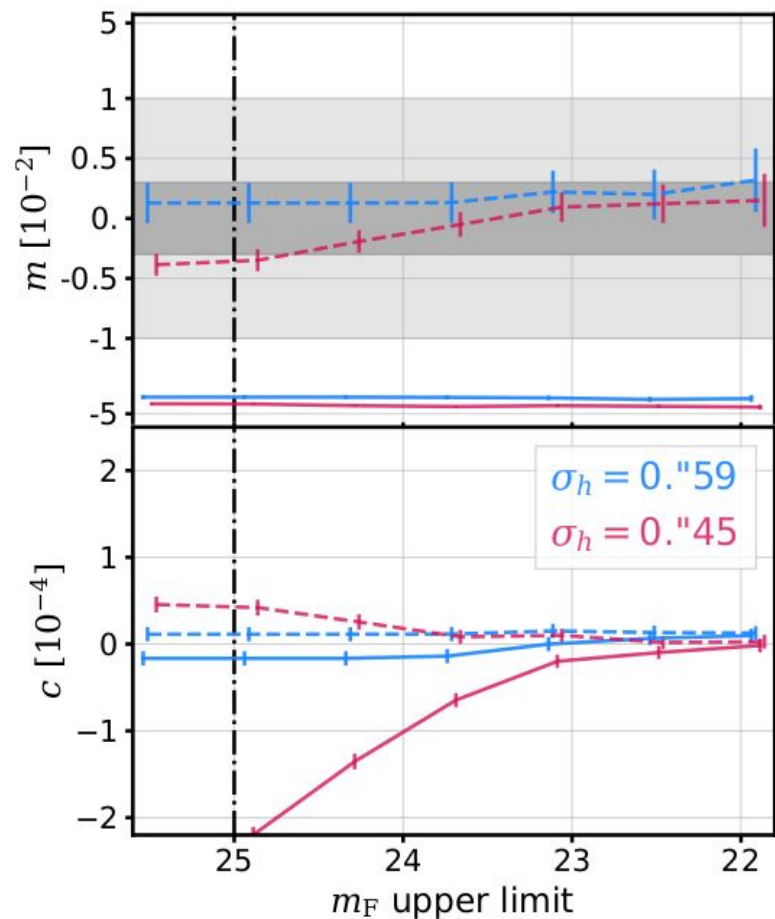


Sim 2:

dashed / solid: with / without analytic bias correction



Sim 3: dashed / solid: with / without analytic bias correction



Summary

1. The analytical shear estimator can control the multiplicative bias **below 1%** even for **blended galaxies**;
2. The analytical shear estimator can process **~1000 galaxies** per cpu second;
3. The analytical shear estimator does not needs creation of multiple catalogs, and is very simple to implement.

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1. Test the performance of the shear estimator with slightly **undersampled images**;
2. Test the algorithm with (multi-bands) **multiple exposures** (e.g., joint detection and joint measurement);
3. Test the algorithm with **inhomogeneous noise** (galaxy source noise).

Thanks!

Codes are available at <https://github.com/mr-superonion/FPFS>

Reference

The following papers are ready to be cited if you find any of these papers interesting or use the pipeline. Comments are welcome.

- [version 3](#): Li & Mandelbaum (2022) correct for detection bias from pixel level by interpreting smoothed pixel values as a projection of signal onto a set of basis functions.
- [version 2](#): Li , Li & Massey (2022) derive the covariance matrix of FPFS measurements and corrects for noise bias to second-order. In addition, it derives the correction for selection bias.
- [version 1](#): Li et. al (2018) build up the FPFS formalism based on [Fourier_Quad](#) and [polar shapelets](#).

Backups: The weighting parameter C

