Analytic Shear Response for X

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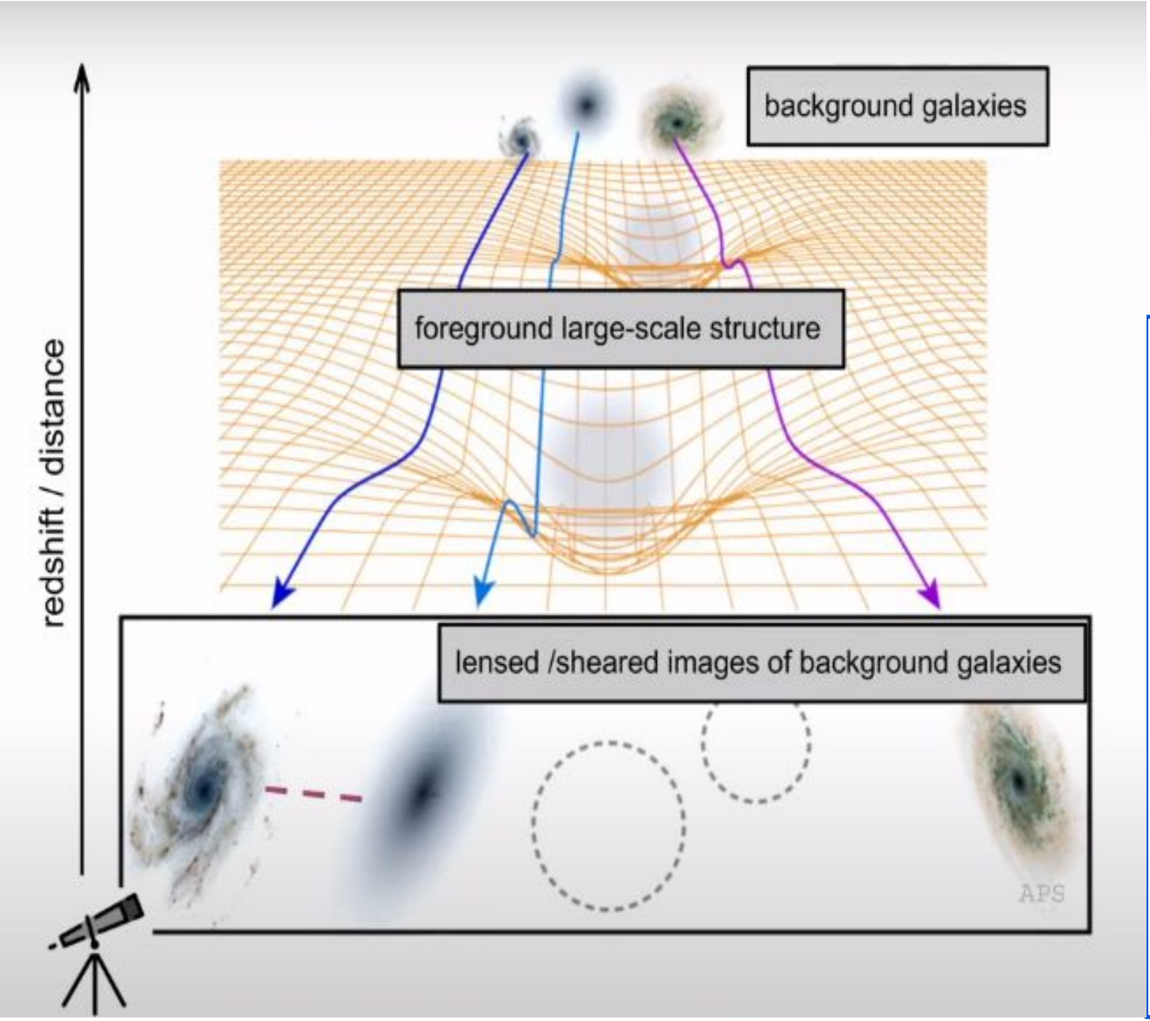
Slack channel desc-wl-shear-fpfs

Two github repos may be useful:
FPFS and impt



$$\mathbf{A} = \begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix}$$

where $g_{1,2} \sim 0.03,\ \sigma_g \sim 0.26$

1. Sensitive to the total mass along the line of sight;
2. Shear is spin-2 and small;

$$\hat{g}_i = \langle R_{ij} \rangle g_j + \mathcal{O}(g^3)$$

3. Shear is a coordinate distortion; no need to know galaxy morphology.

Li & Mandelbaum (2022)

Measuring Shear Distortion

intrinsic gal $\boxed{\bar{f}(\bar{\mathbf{x}})} = \boxed{f(\mathbf{x})}$ lensed gal

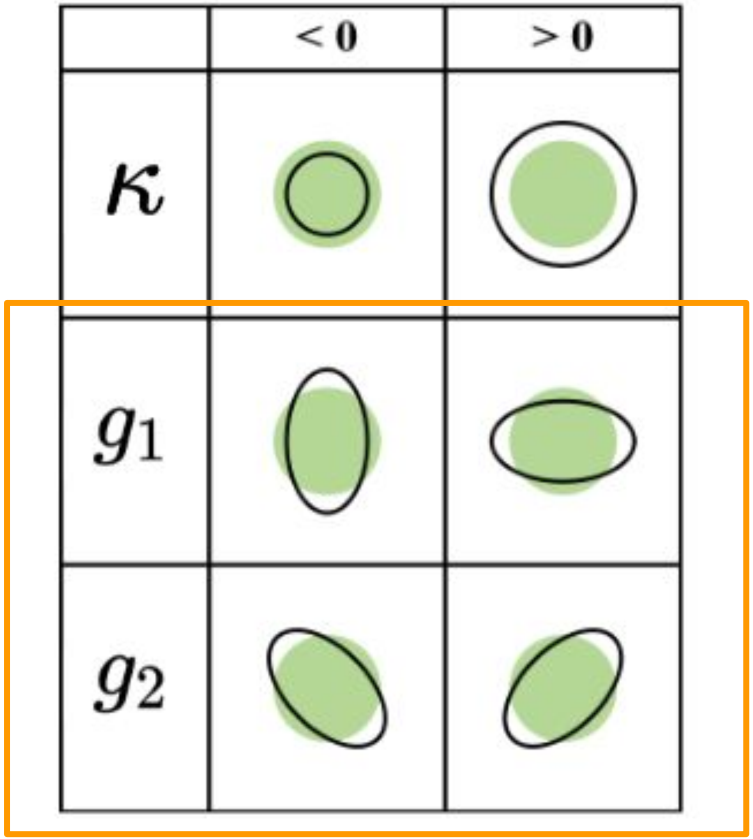
$$\bar{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

$$\mathbf{A} = \begin{pmatrix} 1 - g_1 & -g_2 \\ -g_2 & 1 + g_1 \end{pmatrix}$$

where $g_{1,2} \ll 1$

Assumptions:
(i) intrinsic galaxies are randomly oriented;
(ii) image noise is symmetric to zero.

A simple mathematical problem!
Do we have simple solution to it?



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Metacalibration to estimate linear shear response

We begin from linear observable and will generalize to non-linear observables.

Observable ν: projection of lensed galaxy onto a basis —

$$\nu = \int \underbrace{f(\mathbf{x})}\underbrace{\chi(\mathbf{x})}\mathrm{d}^2x$$

$$= \int \bar{f}(\mathbf{A}\mathbf{x})\chi(\mathbf{x})\mathrm{d}^2x$$

How does ν transform under a shear distortion?

Huff & Mandelbaum (2017), Sheldon & Huff (2017):

$$\nu = \bar{\nu} + g_1 \boxed{\frac{\partial \nu}{\partial g_1}} + g_2 \boxed{\frac{\partial \nu}{\partial g_2}}$$

$$\boxed{\frac{\partial \nu}{\partial g_i}} = \frac{\partial\left(\int \bar{f}(\mathbf{A}\mathbf{x})\chi(\mathbf{x})\mathrm{d}x^2\right)}{\partial g_i}$$

Shear response can be estimated by adding small shear distortions to galaxy image.

A passive line of thought: distortions on bases

$\vec{x} \to \mathbf{J}\vec{x}$

$\mathbf{J} = \mathbf{A}^{-1}$

$$\nu = \int \boxed{\bar{f}(\mathbf{A}\mathbf{x})} \chi(\mathbf{x}) \mathrm{d}^2 x$$

$$\nu \propto \int \bar{f}(\mathbf{x}) \boxed{\chi(\mathbf{J}\mathbf{x})} \mathrm{d}^2 x$$

If we ***fix the galaxy images but distort the bases***, the measurement is **faster**, since we can pre-derive (compute) the **shear responses of the bases**.

$$\frac{\partial \nu}{\partial g_i} \propto \int \bar{f}(\mathbf{x}) \boxed{\frac{\partial \chi(\mathbf{J}\mathbf{x})}{\partial g_i}} \mathrm{d}^2 x$$

Chain rule for derivatives

$$\frac{\partial \chi(\mathbf{J}\mathbf{x})}{\partial g_1} = \left(x \frac{\partial}{\partial x} - y \frac{\partial}{\partial y}\right) \chi(\mathbf{x})$$

$$\frac{\partial \chi(\mathbf{J}\mathbf{x})}{\partial g_2} = \left(x \frac{\partial}{\partial y} + y \frac{\partial}{\partial x}\right) \chi(\mathbf{x})$$

Raising and Lowering Operators (Refregier 2001)

$$\hat{a_1} = x + \frac{\partial}{\partial x} \qquad \hat{a_2} = y + \frac{\partial}{\partial y}$$

$$\hat{a_1}^\dagger = x - \frac{\partial}{\partial x} \qquad \hat{a_2}^\dagger = y - \frac{\partial}{\partial y}$$

$$\boxed{\begin{aligned} \frac{\partial \chi(\mathbf{Jx})}{\partial g_1} &= \frac{1}{2}\left(\hat{a_1}^2 - (\hat{a_1}^\dagger)^2 - \hat{a_2}^2 + (\hat{a_2}^\dagger)^2\right)\chi(\mathbf{x}) \\ \frac{\partial \chi(\mathbf{Jx})}{\partial g_2} &= \frac{1}{2}\left(\hat{a_1}\hat{a_2} - \hat{a_1}^\dagger\hat{a_2}^\dagger\right)\chi(\mathbf{x}) \end{aligned}}$$

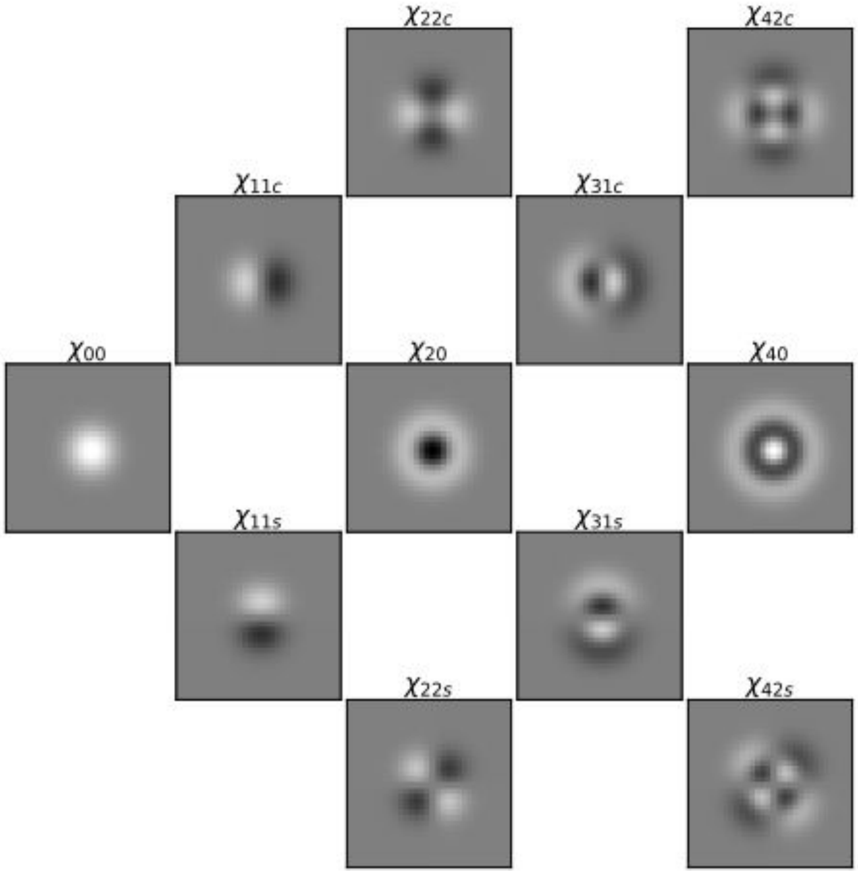
If we use eigen functions of Quantum harmonic oscillator as our basis, shear distortion only couples a finite number of basis.

Shapelet Bases: Eigen functions of 2D QHO

Massey & Refregier (2005):

$$M_{nm} = \int f(\mathbf{x}) \chi^*_{nm} \mathrm{d}^2 x$$

- ‘n' : **radial profile**;
- ‘m' : **spin**;
- ‘c' refers to **real part** (cosine);
- ‘s’ refers to **imaginary part** (sine).



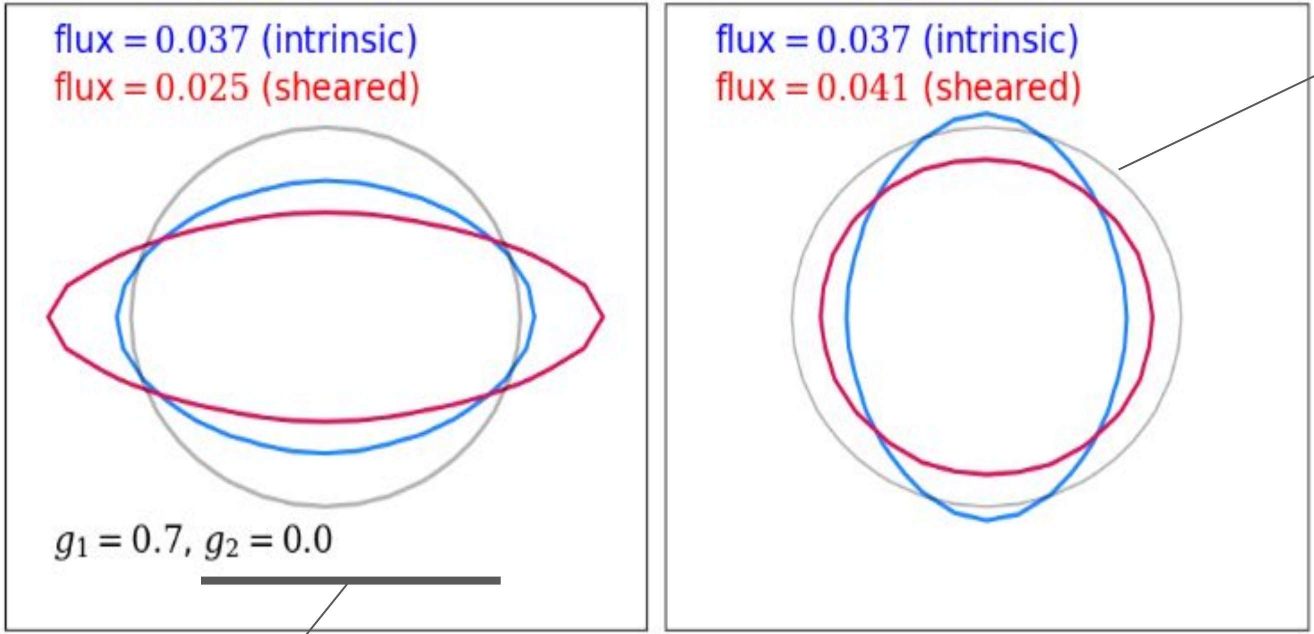
Nonlinear Observable: Ellipticity

$$e_1 + \mathrm{i}\, e_2 \equiv \frac{M_{22}}{M_{00} + C}$$

Anisotropy Caused by Selections

Kaiser (2000)

The following plot gives an example of selection bias caused by a cut on aperture flux



Aperture

As a result, we are more likely to select galaxies that are **anti-aligned** with the shear

We need analytic correction for this !

The shear stretches image in the horizontal direction. The left galaxy (aligned with the shear) appears to be fainter; while the right (anti-aligned with the shear) appears to be brighter.

Selection is a weighting with smooth step function

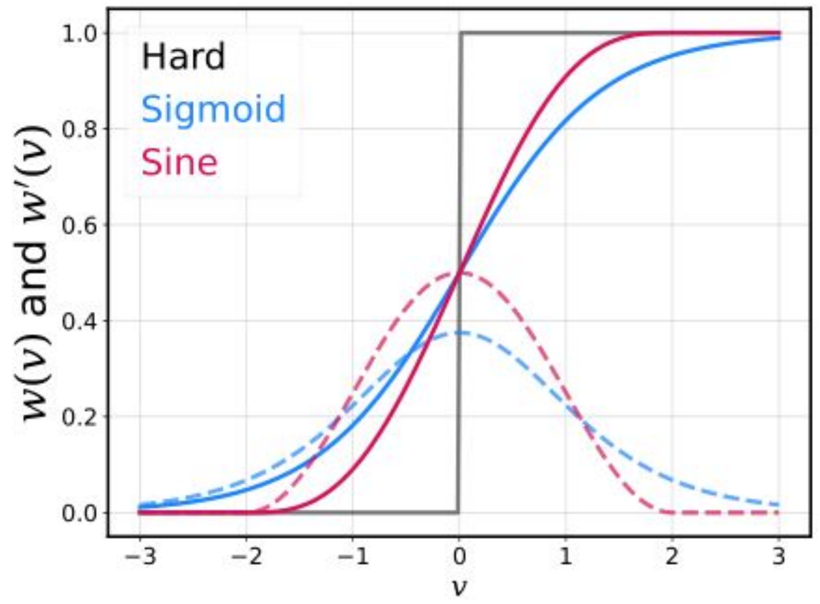
Shear estimation with average ellipticity $\langle w e_\alpha \rangle$

(spin-0) selection weight and (spin-2) ellipticity

$w(\nu_i)$ and $e_\alpha(\nu_i)$

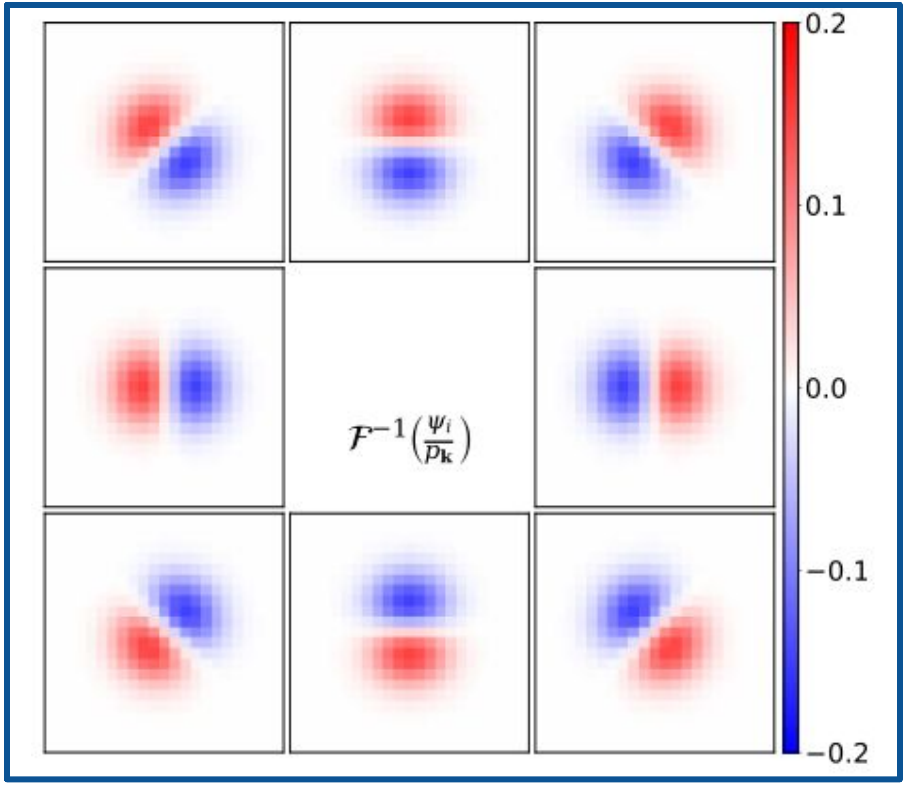
Both the weight and the ellipticity need to be differentiable to get the shear response.

$$w_2(v\,|\,\mu,\omega) = \begin{cases} 0 & \text{if } v \in (-\infty, \mu-\omega) \\ \frac{1}{2} + \frac{(v-\mu)}{2\omega} + \frac{1}{2\pi}\sin\left(\frac{(v-\mu)\pi}{\omega}\right) & \text{if } v \in [\mu-\omega, \mu+\omega] \\ 1 & \text{if } v \in (\mu+\omega, +\infty) \end{cases}$$

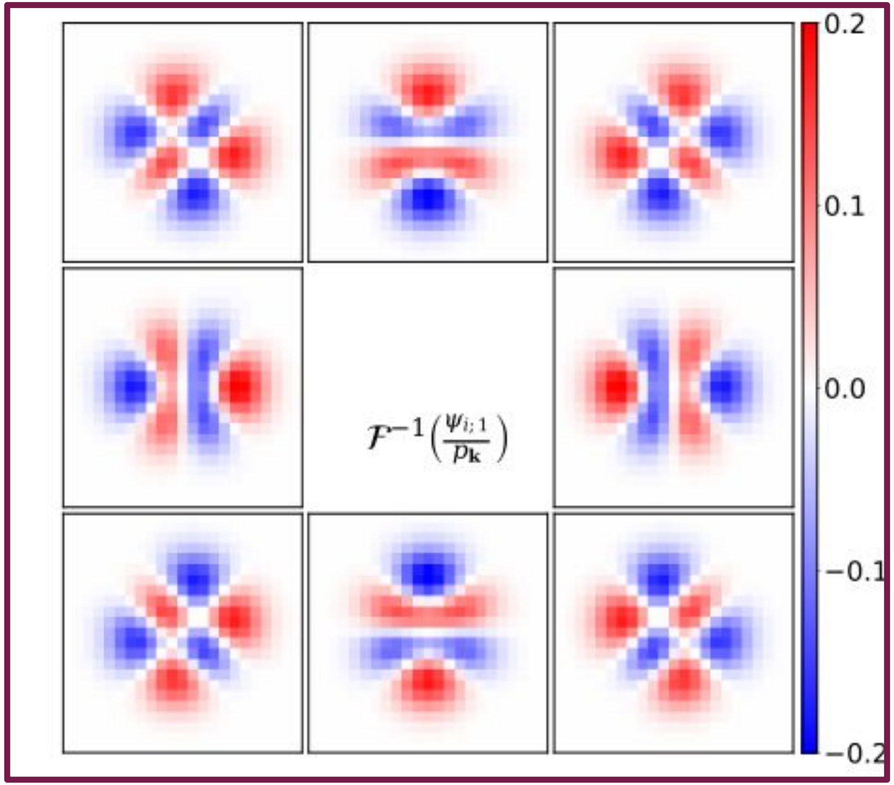


Peak Bases and their Shear Responses

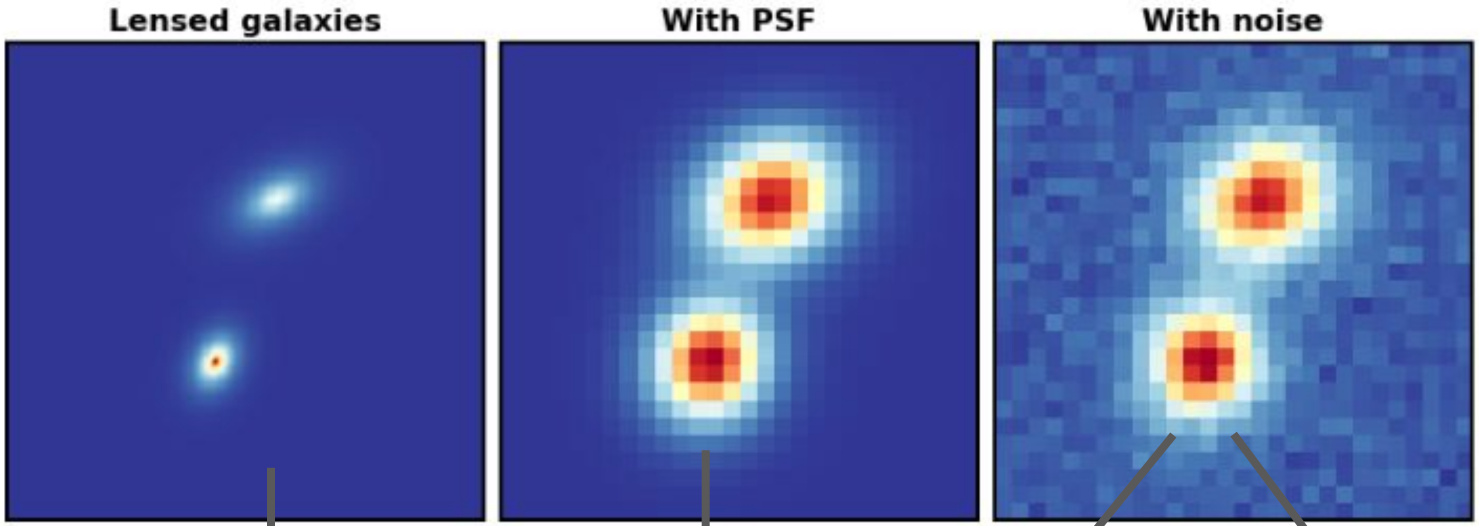
Bases that are used to detect peaks as galaxy candidates



Shear response of those bases.



Systematics in shear measurement



$g \sim 0.03$

$g^2 \sim 10^{-3}$

Lensed gals: ν , e

PSF smearing: $p_{\mathbf{x}}$

noise bias $\langle \tilde{e} \rangle \neq \langle e \rangle$

measurement error: $\delta\nu$, δe

$$\hat{g}_{1,2} = (1 + \boxed{m})g_{1,2} + \boxed{c_{1,2}} + \mathcal{O}(g^3)$$

multiplicative bias **additive bias**

LSST req.

$\delta m < 3 \times 10^{-3}$

PSF revision: Deconvolution in Fourier space

$$f^p(\mathbf{x}) = \int f(\mathbf{x})p(\mathbf{x}-\mathbf{x}')\mathrm{d}^2 x'$$

$$f_{\mathbf{k}} = \frac{f^p_{\mathbf{k}}}{\boxed{p_{\mathbf{k}}}}$$

Fourier transform of PSF

Then project on bases (i.e., shapelet and peak bases):

$$M_{nm} = \int \chi^*_{nm} \frac{f^p_{\mathbf{k}}}{p_{\mathbf{k}}} \mathrm{d}^2 k$$

Note: scale radius of the basis functions should be larger than the scale radius of PSF in configuration space (equivalently, smaller in Fourier space) to avoid noise on small scales being boosted by PSF deconvolution.

Noise is a Perturbation on Galaxy Number Distribution

Noise bias correction:

$$\langle \boxed{\tilde{w}\tilde{e}_\alpha} \rangle = \langle w e_\alpha \rangle + \frac{1}{2}\sum_{i,j} \left\langle \boxed{\frac{\partial^2 (\tilde{w}\tilde{e}_\alpha)}{\partial v_i \partial v_j}} \boxed{\delta v_i \delta v_j} \right\rangle + \mathcal{O}\left((\delta v_i)^4 \right)$$

Noisy observables

Independent of gals

The leading order perturbation is a contraction between Hessian matrix (noise response) and covariance matrix of measurement error on the linear observables.

Under what condition the analytic covariance of measurement errors from image noise can be derived:

	not correlated	correlated
Homogeneous	✅	✅
inhomogeneous	✅	❌

(correlated and homogeneous are defined in configuration space.)

Now in Jax

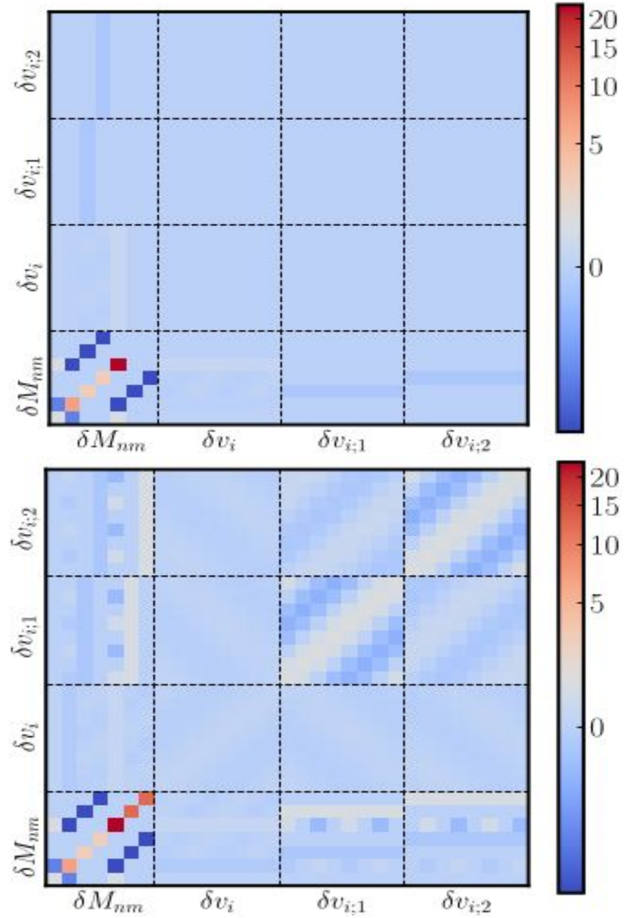
Li et. al (in prep)

Extension of ellipticity definition:

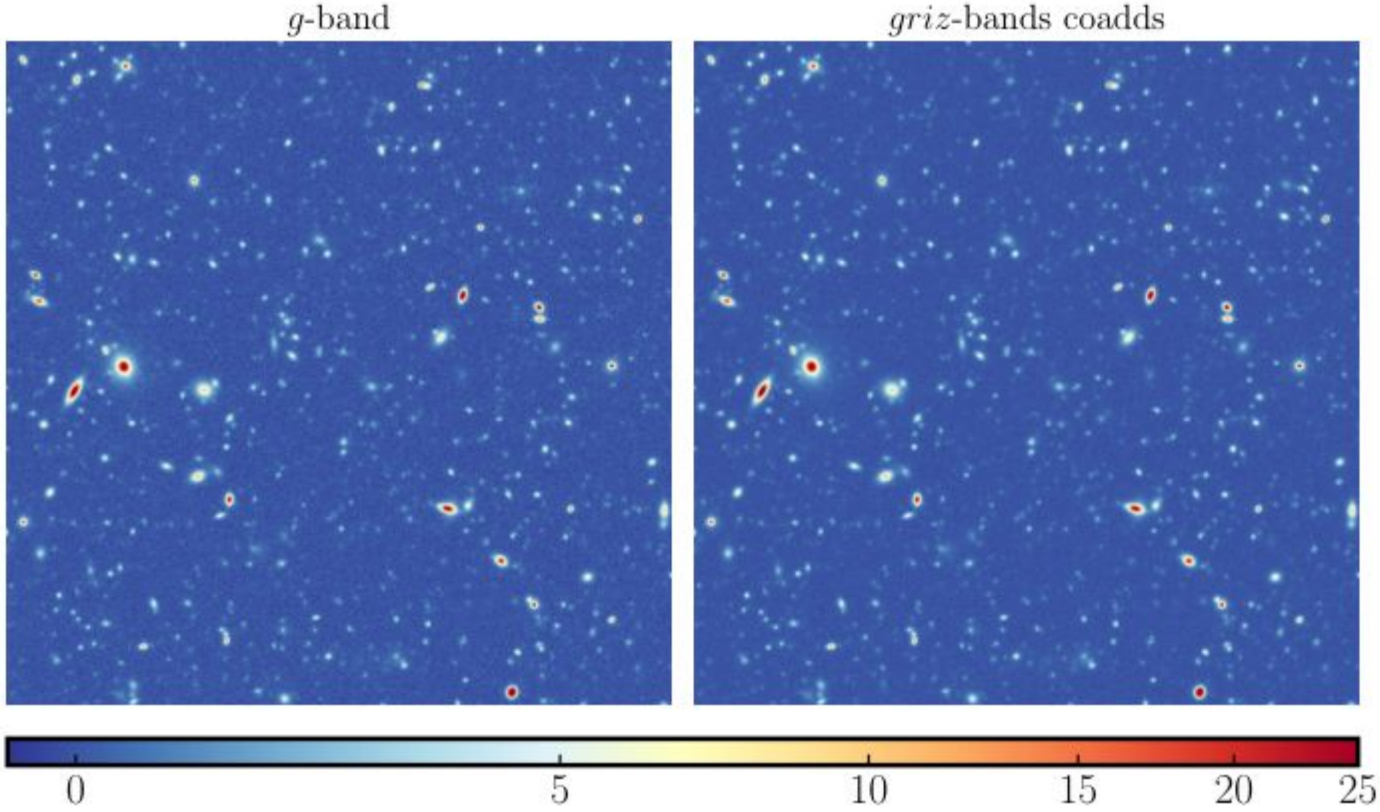
$$e_1 + \mathrm{i}\, e_2 \equiv \frac{M_{22}}{M_{00} + C}$$

$$e_1 + \mathrm{i}\, e_2 \equiv \frac{M_{22}}{(M_{00} + c_0\sigma_0)^{\alpha}(M_{00} + M_{20} + c_2\sigma_2)^{\beta}},$$

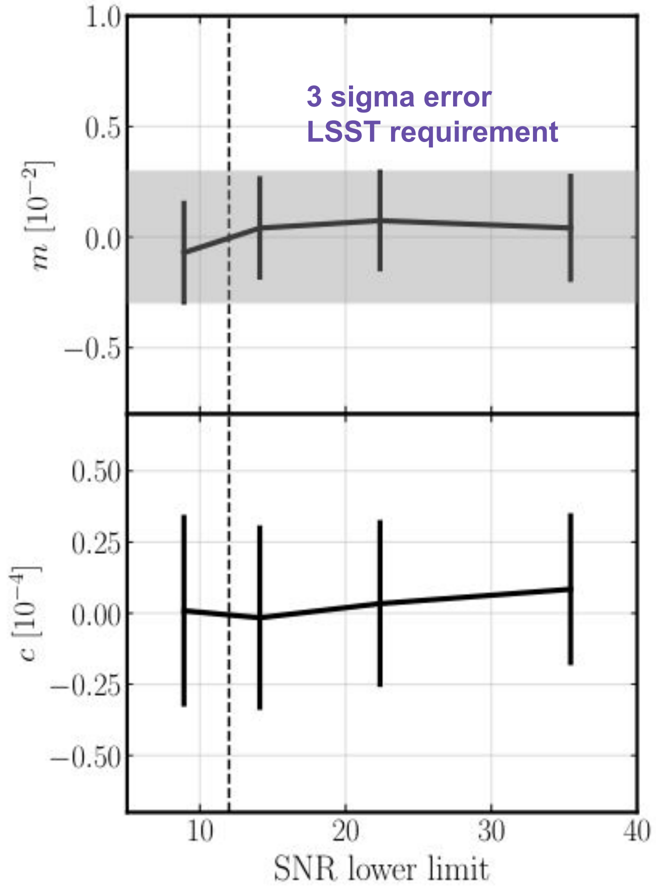
Full noise bias correction:



Test on LSST image simulations (Sheldon et. al (2023))



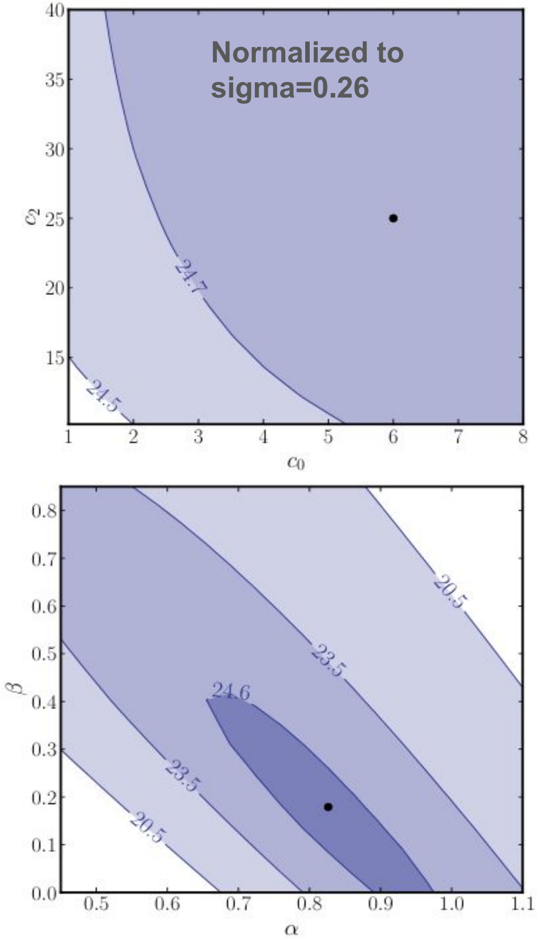
>1000 galaxies per cpu second



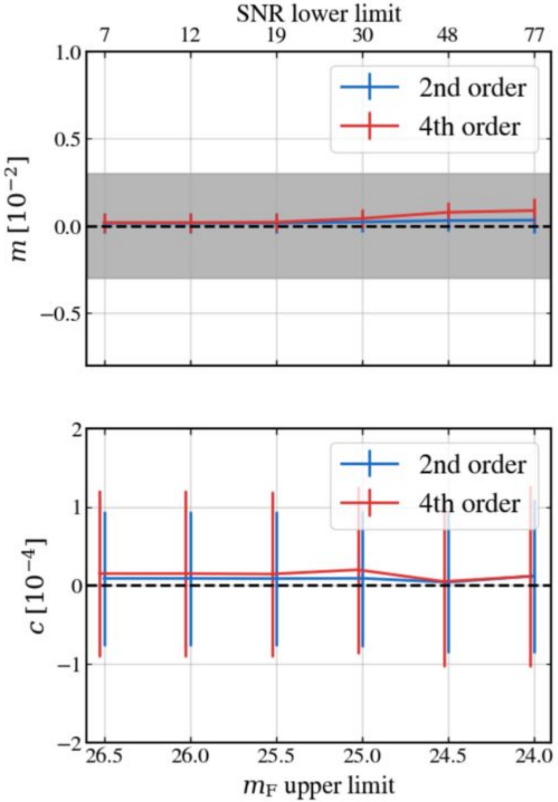
~4k cpu hours (allocation units) to measure shear from all the LSST Y10 images

Effective number density is ~25

Effective galaxy number density



4th order shear estimator



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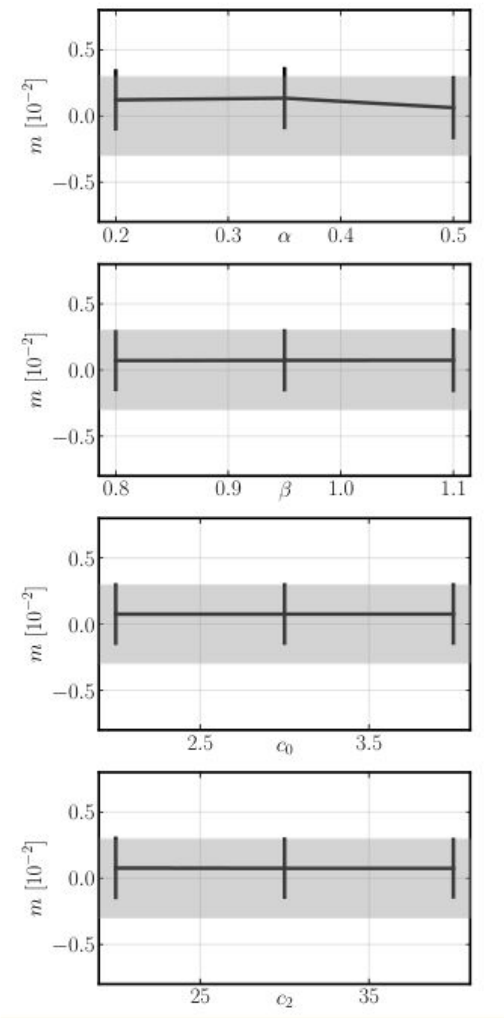
$$e_1 + \mathrm{i}\, e_2 \equiv \frac{M_{22}}{(M_{00} + c_0\sigma_0)^{\alpha}(M_{00} + M_{20} + c_2\sigma_2)^{\beta}},$$

↓

4th order ?



Performance (accuracy) is stable for different hyperparameter setups.

For details see [Li et. al (2018)](); [Li, Li & Massey (2022)](); [Li & Mandelbaum (2023)]();
Li et. al (in prep); Park et. al (in prep)

Three Takeaways:

Analytic shear estimation is simple, accurate, and fast!l

1. Enable us to **directly** understand how different detection, deblending, shape estimation, n(z) influence cosmology constraint;
2. Optimization of constraints from image level;
3. Enable fast joint analysis between Stage-IV surveys.

Future Extension of the Analytic Estimator

Not only a method but a formalism to get analytic shear response for everything

1. Shear response of adaptive shape estimator (Matthew Becker);
2. Shear response of deblending (Arun Kannawadi);
3. Shear response of ???

Question and discussion in DESC slack channel #desc-shear-wl-fpfs
(or make independent channel for each of them?)

Plan in the next 6 months

DESC project
HSC project

1. Test under more realistic simulations with stellar contamination, bad pixel, bright object mask and coadding using descwl-shear-sim.
2. Combine with a differentiable photo-z estimator in jax (e.g. pz-flow) and get the shear response of tomographic photo-z binning.
3. Implement the HSC survey condition in descwl-shear-sim and compare the histogram of galaxy properties between HSCY3 and simulated images.
4. Quantify and calibrate (if larger than HSCY3 requirement) the bias from redshift dependent shear (MacCrann et. al) and apply FPFS to HSCY3.